Chapter 8

FLUID FLOW

GOALS

When you have mastered the contents of this chapter, you will be able to achieve the following goals:

Definitions

Define each of the following terms, and use it in an operational definition:

fluid
density
specific gravity
pressure (absolute and gauge)
buoyant force
streamline flow
viscosity

Fluid Laws

State Pascal's law of hydrostatic pressure, Archimedes' principle of buoyancy, Bernoulli's equation for the conservation of energy in a fluid, and the law of conservation of fluid flow.

Fluid Problems

Solve problems making use of the principles of fluids and conservation laws.

Viscous Flow

Use Poiseuille's law of viscous flow to solve numerical problems.

PREREQUISITES

Before you begin this chapter, you should be able to solve problems that use energy concepts (see Chapter 5).

Chapter 8
FLUID FLOW

8.1 Introduction

Fluids play an important part in our everyday life. The basis of our water and air transportation systems is the buoyancy of objects in water and the lift forces of objects moving through the air. These phenomena are results of fluid dynamics.
The circulation of fluids in our bodies plays an essential part in our energy exchange processes. The circulation of atmospheric gases plays a similar part in the energetics of the earth.

Can you give an example of fluid flow used in energy transfer in your present environment? What is the measure of the flow inertia for a fluid? In this chapter you will be introduced to the basic principles of fluid dynamics. Your understanding of these principles will provide you a basis for analyzing and working with fluid systems.

8.2 Fluids

You are familiar with the properties of fluids. How does a fluid contrast to solid matter?

A *fluid* is a substance that flows easily from one location to another. The term thus applies to both liquids and gases. Liquids and gases differ in several ways. A liquid has a fixed volume but takes on the shape of the container up to the limit of its volume.

A gas takes on both the shape and the volume of its container. Another way in which liquids and gases differ is in compressibility: a gas is easily compressed, and a liquid is practically incompressible - at least for our present consideration.

8.3 Density

There are a number of ways in which substances differ from one another. If you have ever lifted a "brick" made of lead or a small bottle filled with mercury your muscles have felt one striking difference between these two substances and other common materials. Small samples of lead or mercury feel massive. That can be a source of muscular surprise. The*density* of a material, assumed to be homogeneous, is defined as the mass per unit volume. So we use as the defining equation:

$$\rho = m/V \tag{8.1}$$

where ρ is the density, and *m* is the mass of volume V of the material. What are the dimensions of density in terms of M, L, and T? The units are kg/m^3 in the SI system and g/cm3 in the cgs system. What is the numerical factor required to convert kg/m^3 into gm/cm^3?

The *specific gravity* of a substance is the ratio of the density of the substance to the density of water. Hence, specific gravity is dimensionless, that is, a pure number, and is independent of the system of measurement that you use. For example, one cubic foot of lead weighs 705 lb and one cubic foot of water weighs 62.4 lb. Calculate the specific gravity of lead from these data and compare it with the value in Table 8.1.

Material	Density kg/m³	Specific Gravity
Aluminum	2.7×10^3	2.70
Bone	1.85×10^3	1.85
Brass	8.6×10^3	8.60
Copper	8.9×10^3	8.90
Gold	19.3×10^3	19.3
Ice	0.92×10^3	0.920
Iron	7.8×10^3	7.80
Lead	11.3×10^3	11.3
Silver	10.5×10^3	10.5
Steel	7.8×10^3	7.80
Wood (oak)	0.8×10^3	0.8
Ethyl alcohol	0.81×10^3	0.81
Glycerin	1.26×10^3	1.26
Mercury	13.6×10^3	13.6
Water	1.00×10^3	1.00

Table 8.1 Table of Densitites and Specific Gravities

8.4 Force on Fluids

What happens if you push with your finger against the surface of water? Against the surface of ice?

If you investigate the way in which a force acts upon the surface of a liquid and a solid, you find at least one difference. In the case of the solid you can exert a net force in any direction, up, down, or sideways. For an ideal fluid, a fluid with no internal resistance, the net surface force must always be directed at right angles, or normal, to the surface. An ideal fluid at rest cannot sustain a tangential force. The layers of fluid simply slide over one another. An ideal fluid will not maintain a shape of its own. It is the inability of fluids to withstand tangential forces that allows them to flow.

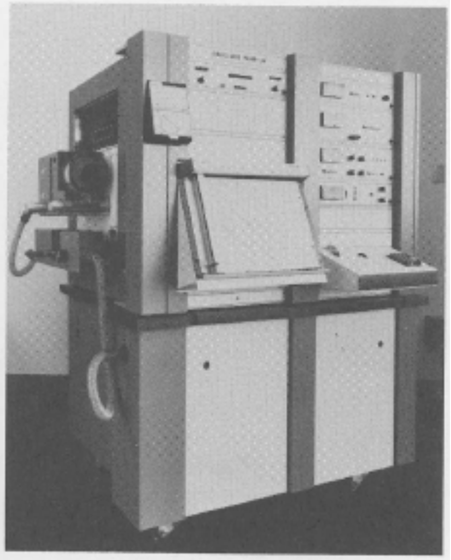
A pressure breathing assistor provides intermittent positive-pressure breathing assistance for patients suffering from chronic bronchiopulmonary diseases such as emphysema, bronchitis, asthma, and other respiratory ailments. (Mine Safety Appliance Company, Pittsburgh, Pa.)

8.5 Pressure

Since a fluid will only sustain a net force perpendicular to its surface, we can speak of a force acting on a fluid in terms of the total force acting on an area of the liquid, or a pressure.

Pressure is defined as the normal force per unit area.

Pressure is transmitted to the interface boundaries and across any section of a fluid at right angles to the boundary or section. Let us consider a fluid under pressure in contact with a surface. Assume that we have a closed surface that contains a fluid, for example, a balloon filled with water (see Figure 8.1).



Pulmonary testing unit. The components of this basic unit are a spirometer, a CO_2 absorber-recorder interface, and an X-Y-T recorder. (Searle Cardio Pulmonary Group—CPI, Houston, Texas.)

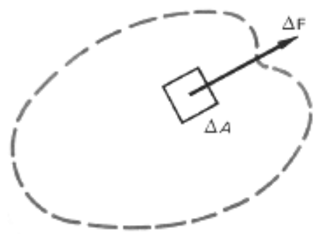


FIGURE 8.1
A closed surface that contains a fluid. ΔA represents the area of a portion of the surface, ΔF is the normal force exerted by the fluid on the surface.

A small portion of the area of the surface may be represented by ΔA. Let ΔF represent the component of the applied force that is perpendicular to the area ΔA. This perpendicular component is called the normal force.

Then we can define the average pressure *P*ave acting on that position of the area as $P_{ave} = \Delta F/\Delta A$. The pressure may depend upon the area chosen. This difficulty can be avoided by choosing an area around a point and having the area decrease and approach the point. Thus, the pressure at a point is defined as the ratio of the normal force to the area, as the area is reduced to a very small size. The pressure at a point can be represented mathematically in the following way:

$$P = \lim_{\Delta A \to 0} (\Delta F/\Delta A) \tag{8.2}$$

The pressure may vary from point to point on a surface. Pressure is a scalar quantity and has the basic units of newtons per square meter (N/m^2) in the SI system, and dynes per square centimeter ($dynes/cm^2$) in the cgs system. In our daily lives we find many other units of pressure in use (centimeters of mercury, for example), but they are reducible to either of the above. Can you think of other pressure units? What are the units of pressure used by your television weather forecaster to measure atmospheric pressure?

8.6 Pressure of a Liquid in a Column

It is a common practice to describe the pressure of the atmosphere by referring to the vertical column of a liquid that the pressure of the atmosphere can support. In this case we are interested in determining the pressure produced by a given height of the liquid in the column shown in Figure 8.2. Let

A = area of *GHH′G′* or *DCC′D′* and let

h = height = *GD* =*HC* = *H'C'* =*G'D'*. For the equilibrium, the force acting on the front *GHCD* is equal in magnitude and opposite in direction to the force acting on the back of the column *G′H′C′D′*. Also the force of $\mathbf{F}_b$ acting on the bottom *DCC′D′* must be equal in magnitude to the force $\mathbf{F}_t$ acting on the top *GG′H′H* plus the weight **W** of the liquid column:

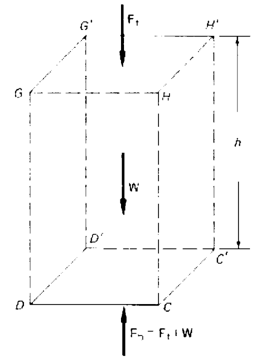


FIGURE 8.2
A rectangular column of liquid. The liquid in the column has a weight **W**. There is a force $\mathbf{F}_t$ acting down upon the top of the column and a force $\mathbf{F}_b$ acting upward on the bottom of the column. The column has a height *h* equal to the length of the edge GD. The top and bottom of the column have the same area *A*, which is equal to the area of the rectangles DD'C'C or GG'H'H.

$$\mathbf{F}_b = \mathbf{F}_t + \mathbf{W}$$

Thus the magnitude of the weight of the liquid column can be expressed as

$$W = F_b - F_t \qquad (8.3)$$

The magnitude of weight is the product of mass times gravitational acceleration, or the product of its density and its volume and *g*, thus

$$W = (\rho V)(g) = (\rho A h)g$$

where *g* is the magnitude of acceleration due to gravity and the volume of the liquid is expressed as the product of its height *h* multiplied by the area of its base *A*. Dividing both sides of Equation 8.3 by the area, (A), this equation becomes

$$(F_b - F_t)/A = \rho g h$$

where $(F_b - F_t)/A$ is the pressure exerted by the column of liquid of depth *h*. We can replace the expression $(F_b - F_t)/A$ by the pressure *P*,

$$P = \rho g h \qquad (8.4)$$

This equation provides an explanation for the use of the variety of units for measuring pressure. The height of a liquid column is directly proportional to the pressure supporting the column. Hence, if you know the height of a supported liquid column you can readily compute the pressure. For example, the standard pressure of the atmosphere at sea level will support a column of mercury 76 cm in height. What is standard atmospheric pressure in N/m^2?

8.7 Gauge Pressure

If the liquid has a free surface point (Figure 8.3) upon which there is a pressure P_0, then the total or absolute pressure at a point at depth h below the surface is given by the sum of P_0 plus the pressure exerted by the liquid above that point.

$$P_T = P_0 + \rho gh \quad (8.5)$$

For example, the pressure at any depth h of a lake, which has an atmospheric pressure on its surface, can be calculated by the above relationship where P_0 becomes equal to atmospheric pressure, P_A. Then, $P_T = P_A + \rho gh$.

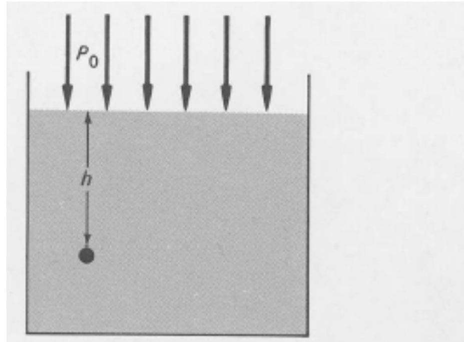


FIGURE 8.3
A liquid in a container with a free surface, upon which a pressure P_0 is acting.

The difference between P_T and P_A is called the *gauge pressure reading*, and in the case of a liquid the gauge pressure at any depth h becomes equal to ρgh. Usually a gauge pressure reading refers to the pressure above atmospheric pressure. For example, if your automobile tire is inflated to a gauge pressure reading of 2.07×10^5 N/m^2 (30 pounds per square inch, psi), the absolute or total pressure in the tire is 2.07×10^5 N/m^2 plus the atmospheric pressure, or about 3.08×10^5 N/m^2 (45 psi).

EXAMPLES

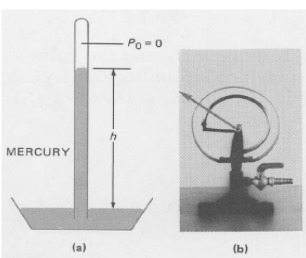


FIGURE 8.4
(a) A mercury barometer. The surface of the mercury in the bowl is open to the pressure of the atmosphere. The pressure on the surface of the mercury in the upper end of the closed tube is zero, except for the vapor pressure of mercury, which can be neglected at room temperature. (b) An aneroid pressure gauge.

1. In a mercury barometer, invented by Torricelli in 1643, the pressure above the mercury in the tube is practically zero except for the mercury vapor pressure which can be neglected at room temperature (Figure 8.4). Hence, the atmospheric pressure is equal to

   $$P_a = \rho_{Hg}\, gh_{Hg}$$

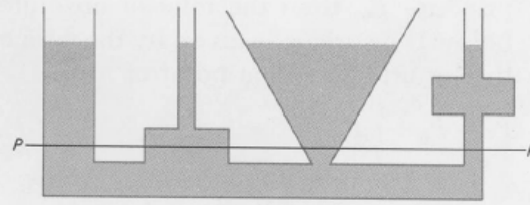
   A barometer reading of 76 cm of Hg is considered standard atmospheric pressure. How long would the tube have to be for a water barometer? Since $\rho_{water}\, gh_{water} = \rho_{Hg}\, gh_{Hg}$ and $\rho_{water} = 1$ gm/cm^3, we see h_{water} = (76 cm)(13.6), or, 10.3 meters.

2. If a number of vessels of different shapes are connected as shown in Figure 8.5, it will be found that a liquid poured into the system will rise to the same height in each. That is, the pressure at all points on a horizontal plane such as PP' will have equal pressures in the vessel. This is consistent with Equation 8.5,

$$P_T = P_0 + \rho g h$$

which states that the pressure depends only on the depth below the liquid surface and not upon the shape of the containing vessel.

FIGURE 8.5
Differently shaped portions of a vessel containing a liquid. The pressure at any point below the surface of the liquid depends upon the depth of the point below the surface of the liquid.



The device for measuring blood pressure is called a sphygmomanometer. The pressure calibration is in mm of mercury. An inflatable cuff is wrapped around the upper arm, and a stethoscope is placed over the artery in the crease of the elbow below the cuff. Before the cuff is inflated, blood is flowing unimpeded, and there is no sound in the stethoscope. As the cuff is inflated, the blood circulation is gradually shut off, and the doctor or nurse hears the thump thump through the stethoscope resulting from pumping of blood by the heart. The cuff is inflated until the listener no longer hears the thump. This means the blood flow has temporarily been shut off by the pressure exerted by the cuff. The pressure is then reduced in the cuff by a controlled release valve, and the observer reads the sphygmomanometer at the exact moment a thump is heard in the stethoscope. This means blood is again flowing in the artery, and the corresponding pressure is called the systolic pressure. The pressure in the cuff is further decreased, and the thump disappears at a lower pressure which is also recorded. This is the *diastolic pressure*. There are many factors that influence the blood pressure reading, and there is a variation among individuals. A typical reading for an adult might be about 140/90, meaning a systolic pressure of 140 mm Hg and a diastolic of 90 mm Hg.

Home use of a sphygmomanometer. Note the height of the mercury column and the use of the stethoscope.

The pressures are produced by the heart. The human heart operates as a muscular pump which on contraction can exert a hydrostatic pressure of about 140 mm Hg (systolic) on the blood. On relaxation of the heart there is still tension in the muscle of the left ventricle to maintain a pressure of about 90 mm Hg (diastolic). The pressure exerted by the heart forces the blood out of the left ventricle of the heart into the aorta and through the blood vessel system. The blood is returned to the right atrium of the heart at very nearly zero gauge pressure.

8.8 Pascal's Principle

We have learned that the pressure at a point within a liquid does not depend upon the shape of the vessel, but only upon the depth of the point below the surface of the liquid. We can use this fact to design a mechanical system to multiply our strength. Consider a system like the one diagrammed in Figure 8.6a, which is filled with an incompressible liquid. If the pressure P_0 is increased in any way, such as by inserting a piston on top of the liquid, the pressure at any depth is increased by the same amount. The principle was originally stated by Pascal in 1653 in the following terms:

A pressure applied to a confined liquid is transmitted undiminished to all parts of the liquid and the walls of the containing vessel.

The above principle is illustrated by a hydraulic press of which Figure 8.6 is a schematic cross-sectional drawing.

A pressure applied to a confined liquid is transmitted undiminished to all parts of the liquid and the walls of the containing vessel.

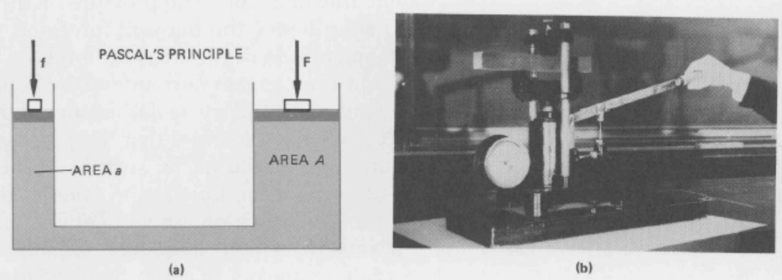


FIGURE 8.6
(a) A closed system filled with an incompressible liquid. The two pistons have different areas. They will support different forces since the pressure exerted by the liquid is the same on both pistons. (b) A laboratory hydraulic press. The scale was reading about 800 lb when the picture was taken. Note the relative size of the two pistons and the mechanical advantage of a lever system. Thus, a relatively small applied force produces a compressive force of 800 lb on the stick of lumber.

We can use Pascal's principle to derive an expression for the load **F** we can lift on the large piston of area A by applying a force **f** on the small piston of area a. The magnitude of the pressure we apply to the small piston is given by the ratio of f to a. When this pressure f/a is applied to a liquid such as oil and is transmitted through the connecting pipe to the large piston where the pressure P must be the same. $P = f/a = F/A$ since by definition the pressure P is equal to F/A. We can use this equation to find the size of F,

$$F = f\,A/a \qquad (8.6)$$

A hydraulic press is a force multiplying device with a theoretical mechanical advantage of A/a. What are other examples of Pascal's principle? Use the conservation of energy to show that the distance the load moves equals $(a/A)L$, where L is distance the applied force moves.

Pascal's principle can be used to explain the comfort of water beds. A water filled mattress applies pressure equally to all parts of a body it supports. The use of water mattresses with chronically ill patients can help prevent bed sores. Can you explain why?

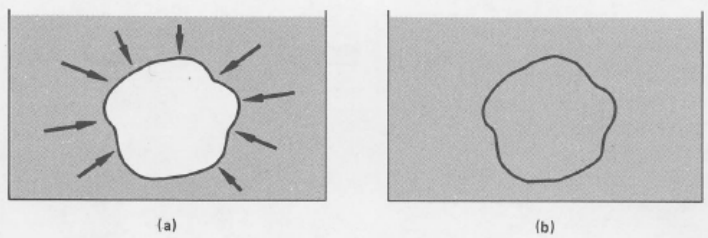
8.9 Archimedes' Principle

You have seen an object floating on the surface of a body of water. Perhaps you have tried to lift a rock in water, and you have found that it is easier (that is, it requires a smaller force) to lift the rock in the water than after it is out of the water. Can you explain this phenomenon?

The explanation is a necessary consequence of the laws of fluid mechanics known as Archimedes' principle. If the body is entirely or partially immersed in a fluid at rest, the fluid exerts a pressure at every point of contact. The resultant of all of the pressure forces is called the buoyant force acting on the body. As the body is at rest, and in equilibrium, the horizontal components of the forces cancel each other, and the weight of the body and the resultant buoyant force must have the same line of action. The pressure on the lower side is greater than on the top side; hence the buoyant force acts upward. We can now determine the magnitude of the buoyant force.

Consider a mass surrounded by a fluid (Figure 8.7a) and think of an imaginary boundary isolating an equivalent amount of fluid (Figure 8.7b).

FIGURE 8.7
(a) A rock submerged in water.
(b) An imaginary shape, the same as the shape of the rock. This shape is assumed to be filled with water.



We have learned that the pressure of the liquid does *not* depend upon the material of the surface. Hence, the buoyant force on the isolated body of the fluid is the same as that on the mass. So we must determine the force on a given volume of the fluid. The fluid, isolated by the imaginary boundary does not move; so the net force acting on it is zero. This means that the buoyant force is equal to the weight of the fluid within the imaginary boundary. Hence, the buoyant force upon the mass is equal to the weight of the displaced liquid.

Archimedes' principle may be stated as follows: *Any object wholly or partially immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid*.

You can determine the specific gravity of a homogeneous solid by the following experiment (Figure 8.8): the body is weighed first in air and then completely immersed in water. W_a = weight in air, W_w = weight in water, W_a - W_w = buoyant force = weight of displaced liquid. The weight of an object in air is given by the product of its density, its volume, and the acceleration of gravity. Since the solid is completely immersed in the water, the volume of displaced water will be the same as the volume of the solid. Hence, the weight of displaced water, which by Archimedes' principle is equal to the buoyant force W_a - W_w, is given by the

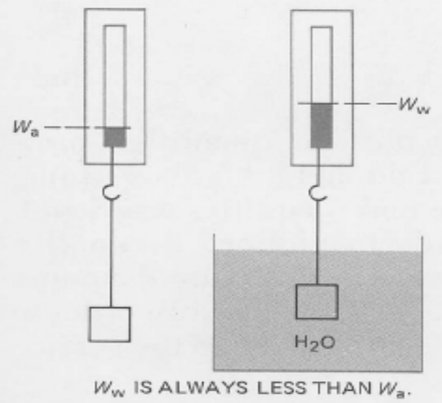


FIGURE 8.8
Using a spring balance, an object is weighed in air and in water. Notice that the weight measured with the object under the water is less than the weight measured when the object was weighed in air. What would the difference be if the object were weighed in a vacuum?

product of the density of water, the volume of the *solid*, and the acceleration of gravity.

$W_a - W_w = \rho_w V g$

where V is the volume of the solid and ρ_w is the density of water. As shown below, the ratio of the weight in air to the buoyant force acting on a completely immersed solid is equal to the specific gravity.

$$Wa/(Wa - Ww) = (\rho_m V g)/(\rho_w V g)$$
$$= \rho_m/\rho_w$$
$$= \text{specific gravity of the solid} \quad (8.7)$$

where ρ_m = density of solid and
ρ_w = density of water.

Can you devise an experiment to determine the density of a liquid? There are at least two ways in which this can be done.

Question

The traditional story is that Archimedes had been asked by a ruler of ancient Greece to determine if his newly purchased crown of gold was really made of solid gold. One evening while in his bath, Archimedes is rumored to have shouted, "Eureka, I have found it!" and to have run to the palace, clad only in his bath towel. Explain how Archimedes had found the crown not to be solid gold. Make up some reasonable numbers, and compute an answer for Archimedes.

8.10 Fluid Flow

Consider the flow of water in the city water mains and the water line when a faucet is opened. Assume a given volume of water is drained and assume streamline flow, which means that every particle of water that goes through point q will also follow through q_1, q_2, q_3,...q_n, where the q's represent points along any line of flow(see Figure 8.9).

FIGURE 8.9
(a) A schematic diagram of the flow of water from the city water mains to a faucet in a private home. (b) In streamline flow, the various portions of the liquid move along nonintersecting lines.

Let the cross-sectional area of the water main be A and the cross-sectional area of the line in the house be a. The volume that flows through the main is the same as the volume which is drained from the faucet and also the same as that which flows through the house line. The volume that flow through a pipe in one second must be equal to the cross-sectional area multiplied by a column length equal to velocity,v^1 in the main, v in the house. The volume of water that flows in one second is given by the product of velocity and area. Since the volume of flow is constant everywhere in the system, the velocity-area product must also be a constant,

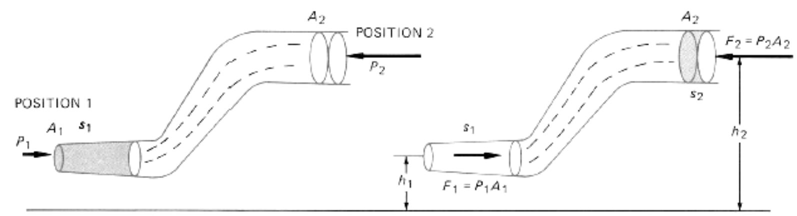
volume of flow per second = $v^1A = va$ (8.8)

This is an example of a conservation law. Note that the velocity of flow will be faster through the smaller pipe than the larger pipe.

8.11 Bernoulli's Theorem

In our discussion of fluids we have used four physical quantities: pressure P, density of fluid, r, the velocity v, and the height h above some reference level. A relationship between these four quantities was developed by Daniel Bernoulli to describe fluids in motion. Bernoulli's theorem can be derived by considering an incompressible liquid moving along a pipe from position 1 to position 2. (See Figure 8.10). In order to move the liquid, we must exert a force F_1 at position 1.

FIGURE 8.10
An incompressible liquid moving along a bent pipe. The cross-sectional area and the elevation of the pipe change from position 1 to position 2 of the pipe.



Then the surface of the liquid near position 1 will move a distance s1. But there may exist a resisting force F_2 at position 2 where the surface of the liquid will move a distance s_2. We can calculate the net work done on the liquid as we cause it to move from position 1 to 2. The net work will be given by the work we did on the liquid at position 1 minus the work done by the liquid against the resisting force at position 2,

net work = F_1s_1 - F_2s_2 (8.9)

since work is the product of force times distance. Remember that the volume of liquid that moves near position 1 must be equal to the volume of liquid that moves near position 2 because the liquid is incompressible. So we can calculate the volume of moved liquid by multiplying the cross-sectional area of the pipe times the distance the liquid surface has moved,

volume of moved liquid = $V = A_1s_1 = A_2s_2$ (8.10)

where A_1 and A_2 are the cross-sectional areas of the pipe at positions 1 and 2 respectively. We can compute the distances the liquid moved at 1 and 2 from the volume of the moved liquid V and the areas, A_1 and A_2,

$s_1 = V/A_1$ and $s_2 = V/A_2$

Substituting these expressions for s_1 and s_2 into Equation 8.9, we obtain the net work as a function of pressure and volume as follows:

net work = F_1s_1 - F_2s_2

net work = F_1V/A_1 - F_2V/A_2 (8.9)

But the ratio of force to area is the definition of pressure, so F_1/A_1 is equal to the pressure at position 1 and likewise F_2/A_2 is the pressure at position 2. Finally then we find the net work is given by the difference between the pressures at positions 1 and 2 times the volume of liquid that moved,

net work = $(P_1 - P_2)V$ (8.11)

From the work-energy theorem we know that the net work that is done on the liquid is equal to the sum of the increase in potential energy and the increase in kinetic energy. As you remember potential energy is energy of position, and kinetic energy depends upon the velocity.

The change in the potential energy between position 1 and position 2 is given by ($mgh_2 - mgh_1$) where m is the mass of the liquid, g is the acceleration of gravity and h_1 and h_2 are the elevations of positions 1 and 2. Equating the net work to the sum of potential-energy change and kinetic-energy change, we find that net work = change in kinetic energy + change in potential energy:

$$(P_1 - P_2)V = (½)(mv_2^2 - mv_1^2) + mgh_2 - mgh_1 \quad (8.12)$$

This equation is a statement of conservation of energy (see Chapters 2 and 4). To obtain an expression that contains the density of the liquid, divide Equation 8.12 by the volume of the moved liquid V. Since the density ρ is given by the ratio of mass to volume, $\rho = m/V$, we obtain

$$(P_1 - P_2) = (½)\rho v_2^2 - (½)\rho v_1^2 + \rho g h_2 - \rho g h_1$$

Rewriting the equation so that all the variables with a subscript 1 are on the left side of the equation and all the subscript 2 variables are on the right, we get

$$P_1 + rgh_1 + (½)rv_1^2 = P_2 + rgh_2 + (½)rv_2^2$$

where the subscripts 1 and 2 refer to the two different positions. Bernoulli's equation may be written simply as follows,

$$P + \rho gh + (½)\rho v^2 = \text{constant} \quad (8.15)$$

where P is the absolute pressure (not the gauge pressure), and r is the mass density. You will note that each term has the dimensions of pressure.

The total pressure in streamline flow is constant. The total pressure is made up of an applied pressure term (P), an elevation term (ρgh), and a velocity term $(1/2)\rho v^2$. You can divide each term in Equation 8.15 by the quantity ρg, and then each term is expressed in units of length. The term $P/\rho g$ is called a *pressure head*, h is an *elevation head*, and $(1/2)v^2/g$ is a *velocity head*. So the total head in streamline flow is a constant,

pressure head + elevation head + velocity head = constant

$$P/\rho g + h + (½)v^2/g = \text{constant} \quad (8.16)$$

Both Equation 8.15 and Equation 8.16 are equivalent ways of expressing the conservation of energy. Conservation of energy is one of the conservation laws introduced in Chapter 2.

Let us examine Equation 8.15, $P + \rho gh + (½)\ \rho v^2$ = constant, more carefully. Consider two points in a horizontal flow, $h_1 = h_2$. Then we have $P_1 + (½)\ \rho v_2^{\ 2}$ = constant. If the velocity at position 1 is greater than the velocity at position 2, the pressure at 1 will have to be less than the pressure at 2 for the equation to remain true. This relationship shows that in a region of higher velocity there is lower pressure.

EXAMPLE

Suppose that you have a spool a pin and a card. The pin is pushed through the card and placed in the end of the hollow spool.(See Figure 8.11). (The only purpose of the pin is to keep the card centered.) You try to blow the card from the spool by blowing into the hollow center of the spool. The air escapes between the end of the spool and the card. This produces an area of low pressure and the atmospheric pressure presses the card to the end of the spool.

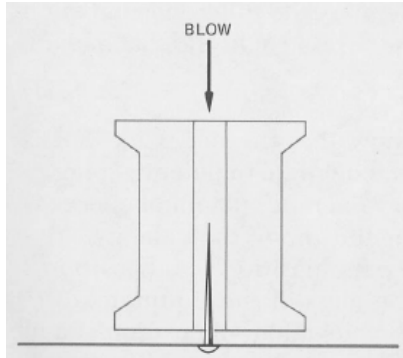


FIGURE 8.11 A spool of thread, a card, and a pin can be used to demonstrate the effects predicted by Bernoulli's equation.

Bernoulli's equation finds application in almost every aspect of fluid flow. Some applications are:

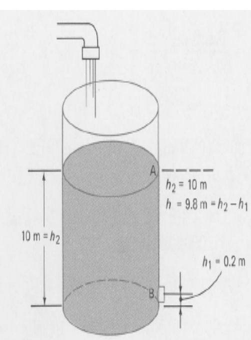
1. the lift on an air foil
2. the operation of an atomizer
3. a curving baseball or golf ball slice
4. the force pushing two passing trucks together on a highway.

(You have undoubtedly noted this last example in your travels.) You should be able to explain each of these phenomena in terms of Bernoulli's equation.

EXAMPLES

1. Consider water maintained at level h_2 (10 m) in the tank and opening at height h_1 (0.2 m) above the bottom of the tank (see Figure 8.12). Find the velocity of outflow at the opening. The pressure at surface A and surface B are each equal to atmospheric pressure. The velocity at A is 0 if a constant height is maintained. Since h_2 is constant and $v_2 = 0$, Bernoulli's equation becomes $\rho gh_2 = (½)\ \rho v_1^{\ 2} + \rho gh_1$ We know also that $h_2 - h_1 = h$, so $(1/2)\rho v_1^{\ 2} = \rho gh$ v_1 = SQR RT [2gh] = SQR RT [2(9.80)(9.80)] = 13.8 m/sec

FIGURE 8.12 A tank with an orifice 0.2 m above the bottom of the tank. Water is kept at a level of 10 m above the bottom.



2. Water flows through the pipe shown in Figure 8.13 at a rate of 120 liters per second. The pressure at position A is 2.00 x 10^5 N/m ^{2}. The cross- section of position B is 60.0 cm^2, and the cross section of position A is 100 cm^2. What is the velocity at A and at B? What is the pressure at B?

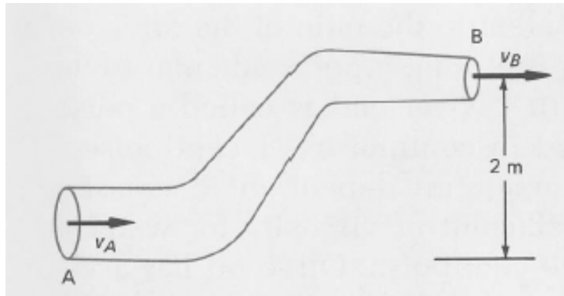


FIGURE 8.13
A pipe of varying size is used to transport water.

We can let Q be the volume flow rate as follows:

$Q = Av = 120 \times 10^{-3}\ m^3/sec = A_1v_1 = A_2v_2$

$0.120\ m^3/sec = 100 \times 10^{-4} m^2 \times v_1$

$v_1 = (0.12\ m^3/sec)\ /\ 10^{-2}\ m^2 = 12.0\ m/sec$

$v_2 = (0.120\ m^3/sec)\ /\ 60.0 \times 10^{-4}\ m^2 = 20.0\ m/sec$

$P_1 + (½)\rho v_1^2 = P_2 + (½)\rho v_2^2 + \rho gh$

$(2.00 \times 10^5\ N/m^2) + (½)\ (1000\ kg\ m^3)(12.0\ m/sec)^2$

$= P_2 + (½)(1000\ kg\ m^3)(20.0\ m/sec)^2 + (1000\ kgm^3)(9.80m/sec^2)(2.0m)$

$P_2 = 2.00 \times 10^5\ N/m^2 + (½)\ (1000)(12)^2\ N/m^2 - (½)\ (1000)(20)\ N/m^2 - (2000)(9.8)\ N/m^2$

$= (2.00 \times 10^5 + 72 \times 10^3 - 200 \times 10^3 - 19.6 \times 10^3)\ N/m^2$

$= 2.00 \times 10^5\ N/m^2 - 1.476 \times 10^5\ N/m^2 = 5.24 \times 10^4\ N/m^2$

8.12 Poiseuille's Law of Viscous Flow

Bernoulli's equation is applicable for fluid flow cases in which there is no friction. We will now consider the case of viscous flow where friction must be considered. A diagram of a section of a tube in which there is a flowing viscous liquid is shown in Figure 8.14. The dotted line represents a transverse plane. The vectors represent the velocity of the liquid in the tube. An analysis of liquid flow of this type shows that the velocity of flow varies from maximum at center of the tube to minimum at the wall, $v = v(r)$, that is, v is a function of r. The velocity is constant for a given distance r from the center of the tube throughout the length of the tube. The velocity of liquid flow is zero at the wall for nonturbulent flow, and there is a frictional force $\mathbf{F}_f$ that opposes motion since the liquid is viscous. Calculus methods can be used (see Section 8.14) to derive the expression for the velocity of liquid flow as a function of the distance r from the center of the tube, the radius of the tube R, the pressure gradient along the tube $(P_1 - P_2)\ /L$, and the viscosity of the liquid h,

$$v = [(P_1 - P_2) / 4\eta L] (R^2 - r^2) \qquad (8.17)$$

where L is the length of the tube, $P_1 - P_2$ is the difference in pressure between the ends of the tube, η is the coefficient of viscosity of the liquid, and R is the radius of the tube. The coefficient of viscosity is a relative measure of liquid friction and is equivalent to the ratio of the force per unit area to the change in velocity per unit length perpendicular to the direction of flow. A viscosity of one 10^{-5} N-sec/cm^2 is called a poise. Small viscosities are usually expressed in centipoises (1 centipoise = 10^{-2} poise). The viscosity of fluids is temperature dependent, decreasing with increasing temperature. The coefficient of viscosity for water at normal room temperature is about one centipoise. Olive oil has a viscosity of about 100 times that of water, and the viscosity of castor oil is about 1000 times that of water. The viscosity of whole human blood is about four times the viscosity of water.

The volume of flow per second (rate of flow) through a tube is given by the product of the average velocity of the liquid flow times the cross-sectional area A of the tube,

rate of flow = average liquid velocity x area = $v_{ave}A$ (8.18)

The velocity of liquid flow varies from zero at the tube wall to $R^2(P_1 - P_2)/4\eta L$ at the center of the tube. The average velocity of liquid flow is one-half of the velocity at the center of the tube. Since the area of the tube is given by πR^2, we can obtain an equation that relates the rate of flow in m^3/sec of a viscous liquid to the pressure gradient, the radius of the tube, and the viscosity of the liquid,

rate of flow (m^3/sec) = $v_{ave}A$

$= (1/2) [(P_1 - P_2) / 4\eta L] R^2 \pi R^2$

$= \pi/8\eta [(P_1 - P_2)/L] R^4$ (8.19)

This equation is the law for viscous flow derived by Poiseuille (pronounced Pwazswee). Poiseuille's law shows that the rate of flow of a viscous liquid is proportional to the pressure gradient $(P_1 - P_2)/L$, inversely proportional to the viscosity of the liquid h, and proportional to the fourth power of the radius R of the tube.

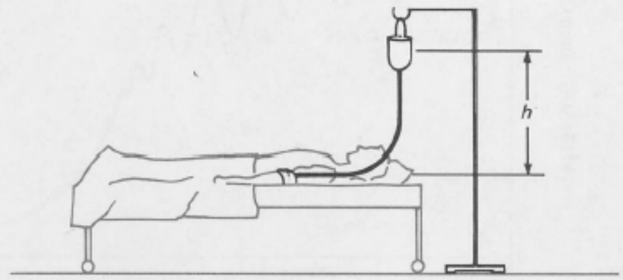
Even though blood vessels are neither circular nor rigid and even though the coefficient of viscosity of blood varies with the pressure gradient, Poiseuille's law is useful in considering the rate of blood flow along the vessels of blood circulation.

EXAMPLES

1. The flow of a liquid through a hypodermic needle is an example of an application of Poiseuille's law of viscous flow. Note that the rate of flow depends upon the fourth power of the radius of the needle and the first power of the pressure gradient (i.e., force exerted on the plunger).

2. Consider the flow of intravenous (I.V.) fluids as administered in a hospital. A typical I.V. arrangement is shown in Figure 8.15. What variables of this system will control the flow of the fluid? Why is the I.V. bottle suspended above the patient? If the I.V. needle is reduced to half its original size, how high would you have to raise the I.V. bottle to keep the flow rate constant?

FIGURE 8.15
Administration of intravenous fluid.



8.13 Blood Flow in the Human Body

The blood circulation system in the human body is a closed system. The continuous flow of blood is caused by a circulating pump, the human heart. The heart works to force blood to flow through the human system as shown in Figure 8.16. The pressure of the blood at various parts of the circulatory system is shown in Figure 8.17. The average pressure of the blood as it enters the aorta is 100 mm Hg. The blood pressure has dropped to 97 mm Hg as the blood, leaving the aorta, enters the arteries. The blood pressure at the entrance to the arterioles is 80 mm Hg. The pressure difference across the arterioles is about 55 mm Hg. So the blood pressure at the entrance to the capillaries is only 30 mm Hg. Of that pressure 20 mm Hg is lost in transit through the capillaries. Thus the blood pressure in the veins is not more than 10 mm Hg.

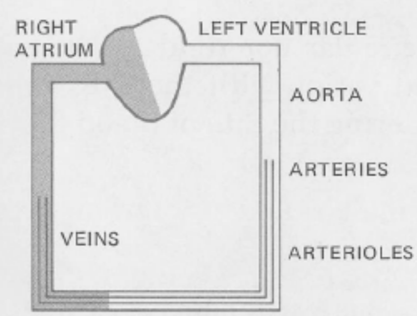
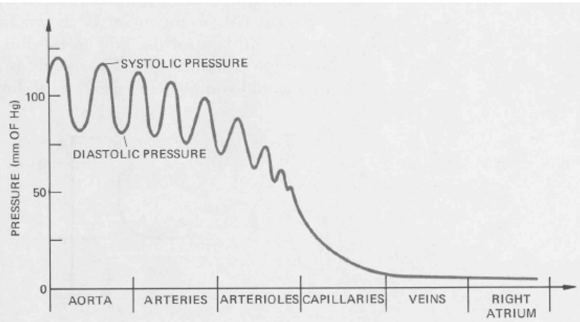


FIGURE 8.16
A schematic diagram of the human circulatory system.

FIGURE 8.17
The blood pressure (mm Hg) in various parts of the circulatory system.



An interesting property of human blood flow is the total quantity of blood that flows through the human system. A typical value for the speed of blood flow through the aorta of an adult is about 35 cm/sec. The aorta, which has a diameter of about 1.8 cm, has a cross-sectional area of about 2.5 cm^2. The total volume of blood flow is the product of the velocity times the area (see Equation 8.18):

quantity of flow = (35 cm/sec)(2.5 cm^2) = 88 cm^3/sec

So in a typical human being, about 100 cm^3 of blood passes through the system in one second. Of course, during periods of physical exercise the total flow of blood increases. This increase is accomplished by an increase in the blood pressure and by a decrease in the resistance of the circulatory system to the flow of blood. This decrease in resistance is caused by the dilation of the blood vessels, that is, an increase in the diameter of the vessels. Since the flow of blood is proportional to the fourth power of the diameter of the blood vessels, a small increase in the vessel diameters can produce a substantial increase in the rate of blood flow. In a similar way, anything that decreases the effective diameter of the blood vessels will cause the heart to work harder than normal to maintain a constant rate of blood flow. We can calculate the rate at which the human heart does work from the product of the force exerted by the heart F and the velocity of the blood v (see Chapter 5, Equation 5.12),

$$\text{power} = Fv \qquad (5.12)$$

Where F is the average force acting on the blood and is given by the blood pressure times the area of the aorta and where v is the velocity of the blood flow and is given by the ratio of the rate of blood flow divided by the area of the aorta. Hence, we can substitute these quantities into Equation 5.12 to show that the power output of a human heart is given by the product of the blood pressure times the rate of blood flow,

$$\text{power} = Fv = \text{pressure x velocity of flow} \qquad (8.20)$$

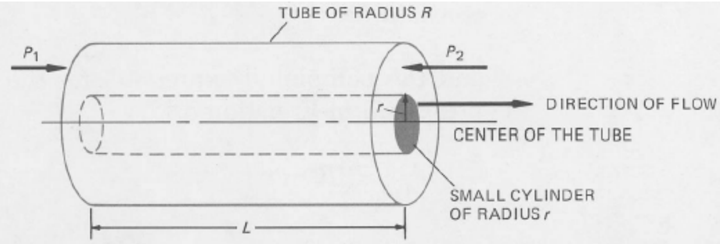
An average blood pressure in a human is about 100 mm Hg (1.3 x 10^4 N/m^2), and the quantity of flow is on the order of 100 cm^3/sec (1 x 10^{-4} m^3/sec). So we find that the normal power output of the heart is 1.3 watts, which is about one percent of the total power dissipated by the human body.

ENRICHMENT 8.14 Viscous Flow

When considering the flow of a viscous liquid through a tube, we know that the velocity of the liquid will vary from one point in the tube to another. For example, we know that the viscous liquid will tend to stick to the inside wall of the tube, so the velocity of flow of the liquid will be small near the wall. In order to find a mathematical expression for the flow of a viscous liquid through a tube, let us make the assumption that the velocity of flow is maximum at the center of the tube, that the velocity at the wall is zero, and that the velocity of flow can only be different at different distances from the center of the tube (see Figure 8.14).

The coefficient of viscosity is defined as the ratio of the magnitude of the force required to slide along a unit area of liquid to the velocity gradient in a direction perpendicular to the direction of flow. Let us consider a small cylinder of liquid (radius r) inside of the tube (radius R and length L) as shown in Figure 8.18.

FIGURE 8.18
A tube of radius R and length L transmitting a viscous liquid from a pressure of P_1 to a lower pressure P_2.



The force acting to slide this small cylinder of radius r in the direction of flow is given by the difference between the pressures acting on the ends of the cylinder times the area of the end of the cylinder,

force = $(P_1 - P_2)\pi r^2$ (8.21)

The surface area of liquid that is being pushed along by this force is just the outer surface area of the small cylinder,

surface area of the cylinder = $2\pi rL$ (8.22)

Hence, the force acting per unit area to slide the liquid along is given by the ratio of these two equations, Equation 8.21 and Equation 8.22,

force per unit area = $(P_1 - P_2)\pi r^2/2\pi rL = (P_1 - P_2)r/2L$ (8.23)

We can put this expression into the equation that defines the coefficient of viscosity and obtain an expression that will relate the velocity gradient to the radius of the small cylinder:

η = force per unit area/velocity gradient = $[(P_1 - P_2)r/2L]/(-dv/dr)$ (8.24)

where $-dv/dr$ is the velocity gradient in the direction perpendicular to the direction of flow and h is the viscosity. The minus sign in front of dv/dr indicates that the velocity decreases as the value of r increases. In order to find an expression for the velocity of flow as a function of the distance from the center of the tube, we can rearrange the terms in Equation 8.24 to separate the variables v and$r$. Then we integrate both sides of the equation, as follows:

$\int dv = -\int\{(P_1 - P_2)r/2\eta L\}\, dr$ (8.25)

$v = -\{(P_1 - P_2)r^2/4\eta L\}$ + constant (8.26)

To evaluate the constant in Equation 8.26, we require that the velocity of flow be zero at the wall of the tube where $r = R$:

$v = 0 = [-\{(P_1 - P_2)r^2/4\eta L\}$ + constant

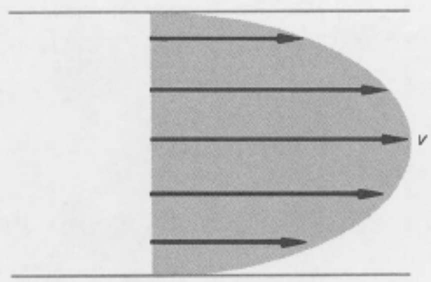
when $r = R$. Thus the constant has the value constant = $[+\{(P_1 - P_2)R^2/4\eta L\}$

and the completed expression for the velocity of flow is the same as presented in Equation 8.17.

$$v = \{(P_1 - P_2)/4\eta L\}\,[R^2 - r^2] \qquad (8.17)$$

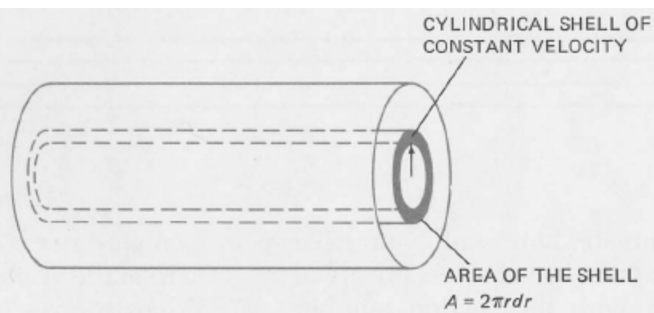
This expression gives a parabolic radial distribution for the flow of a viscous liquid(see Figure 8.19).

FIGURE 8.19
The parabolic distribution of flow velocities for a viscous liquid under conditions of streamline flow.



We can use the expression for the radial dependence of velocity to derive Poiseuille's law. You may recall that the rate of flow is given by the velocity of flow times the area. Since the liquid flow velocity depends only on the distance of the liquid from the center of the tube, all portions of the liquid at the same distance from the center of the tube are moving with the same velocity. In other words, the liquid moves along as if it were thin cylindrical shells sliding inside of one another (see Figure 8.20).

FIGURE 8.20
A viscous liquid under conditions of streamline flow contains cylindrical shells moving at constant velocity.



Each cylindrical shell has a different value of flow velocity as given by Equation 8.17. The area of the end of the shell of radius *r* is the circumference of the shell ($2\pi r$) times the shell thickness (dr),

$$\text{area} = 2\pi r\, dr \qquad (8.27)$$

Hence the rate of flow for each shell is given by the product of velocity times area,

$$\text{rate of flow for a shell} = v \times \text{area} \qquad (8.28)$$

$$= [(P_1 - P_2)/4\eta L]\,(R^2 - r^2)\,2\pi r\, dr \qquad (8.29)$$

by substitution from Equations 8.17 and 8.27. The total flow rate for all of the liquid in the tube is the sum of the flow for all the shells; so we integrate Equation 8.29 from $r = 0$ to $r = R$,

rate of flow $= 2\pi\,[(P_1 - P_2)/4\eta L] \int_0^R (R^2 - r^2)r \mid dr$ (8.30)

$= \pi\,[(P_1 - P_2)/2\eta L]\,[R^2r^2/2 \mid^R_0 - r^4/4 \mid^R_0$ (8.31)

$= \pi\,[(P_1 - P_2)/2\eta L]\,(R^4/2 - R^4/4)$ (8.19)

$= (\pi/8\eta)\,[(P_1 - P_2)/L]\,R^4$ (8.19)

which is Poiseuille's law of viscous flow.

SUMMARY

Definitions

Answer questions 1 to 7 from the definitions of the following terms:

fluid	gauge pressure
density	buoyant force
specific gravity	streamline flow
absolute pressure	viscosity

1. A rigid, boxlike object with dimensions of 12 cm x 12 cm x 8 cm has a mass of 3.4 kg. This object is not a fluid because ______________.
2. The object in question 1 will ___________ (*float*/*sink*) in water because
   a. it is made out of plastic
   b. it is heavier than air
   c. it is lighter than water
   d. it has a density greater than one
   e. it has a specific gravity greater than one
   f. it has a density less than 10^3 kg/m^3
   g. it has a specific gravity greater than 1 g/cm^3
3. If you add the magnitude of the __________ pressure to the value of the __________ pressure, you will obtain the __________ pressure. Therefore, for the normal systems the __________ pressure is always a smaller number than the __________ pressure.
4. Explain why objects weighed in air and in a vacuum do not weigh the same.
5. If a liquid does exert a buoyant force on an object placed in it, how can you determine if an object will sink or float in the liquid?
6. Compare streamline flow with the assumptions made to treat the viscous flow of a liquid in Section 8.10.

7. Viscosity may be considered as an analog to what property of a mechanical system?
   a. potential energy
   b. kinetic energy
   c. actual mechanical advantage
   d. theoretical mechanical advantage
   e. power output
   f. friction
   g. torque
   h. perspicacity
   i. virtuosity

Fluid Laws

8. Group together into sets the letters that represent concepts or words that belong together, and explain the common features of the set. You may put a term in more than one set.

a. Pascal
b. Archimedes.
c. Bernoulli
d. hydrostatics
e. incompressible fluids
f. streamline flow
g. ρgh
h. specific gravity
i. conservation of energy
j. conservation of fluid flow
k. hydraulic press
l. buoyancy
m. golden crown
n. vA = constant
o. $P + \rho gh + (½)\ \rho v^2$ = constant
p. $(½)\ kA^2 + (½)\ mv^2$ = constant
q. a curving baseball
r. a hot-air balloon

Fluid Problems

9. A hydraulic jack which is designed to lift the head of a hospital bed has a small piston of diameter 1.5 cm which is used to apply pressure to a liquid. The liquid then applies pressure to a large cylinder of 9.0 cm in diameter. What is the ratio of the force applied by a nurse to the small cylinder to the load lifted by the large cylinder?

   a. 6.0 b. 0.17 c. 0.028 d. 36.0 e. 13.5

10. What similarities and differences will you notice if you go swimming in fresh water (specific gravity = 1.00), in the ocean (sp. gr. = 1.03), the Great Salt Lake (sp. gr. = 1.12), and the Dead Sea (sp. gr. = 1.18)?

11. Water is flowing through a closed pipe system, and at one point the velocity of flow is 10.0 m/sec while at a point 30.0 m higher the velocity of flow is 12.5 m/sec. If the pressure at the lower point is 4.00×10^5 N/m^2 (a) what is the pressure at the upper point, and (b) if the water flow stops, what is pressure at the upper point?
    a. 4.00×10^5 N/m^2, 4.00×10^5 N/m^2
    b. 4.00×10^5 N/m^2, 6.94×10^5 N/m^2
    c. 4.00×10^5 N/m^2, 1.06×10^5 N/m^2
    d. 7.78×10^4 N/m^2, 1.06×10^5 N/m^2
    e. 7.78×10^4 N/m^2, 4.00×10^5 N/m^2
    f. 1.06×10^5 N/m^2, 7.78×10^4 N/m^2

Viscous Flow Problem

12. An elderly heart patient with hardening of the arteries has the effective diameter of his blood vessels reduced by 16 percent. What is the reduction of blood flow at constant pressure, and by what factor must his blood pressure increase if the rate of flow is to remain constant?
    a. The flow would decrease by 16 percent; so his pressure must increase by 16 percent.
    b. The flow would decrease by a factor of 0.71; so his blood pressure would have to increase by a factor of 1.42.
    c. The flow would decrease by 41 percent; so the blood pressure would have to increase by 59 percent.
    d. The flow would decrease by a factor of two; so his blood pressure would have to increase by a factor of two.
    e. The flow would decrease by 84 percent, and the pressure would increase by 16 percent.

Answers

1. It is rigid. (Section 8.2)
2. Sink, e (Section 8.3)
3. atmospheric, gauge, absolute, gauge, absolute (Section 8.7)
4. because the buoyant force exerted upon objects weighed in air reduces their apparent weight (Section 8.9)
5. If its weight is greater than the buoyant force acting on it, the object will sink. Can you rephrase this statement in terms of density or specific gravity using Archimedes principle? (Section 8.9)
6. The viscous flow in Section 8.12 is treated as if it were streamline flow.
7. f (Section 8.12)
8. a, d, e, k (Section 8.8); b, h, l, m, r (Section 8.9); c, f, i, o, q (Section 8.11); j, n (Section 8.10)
9. d (Section 8.8)

10. The buoyant force is the same in all cases (i.e., equal to your weight), but less liquid is displaced as you swim in liquid of greater specific gravity. You float with a larger portion of your body above the surface of the liquid (Section 8.9)
11. d (Section 8.11)
12. d (Section 8.12).

ALGORITHMIC PROBLEMS

Listed below are the important equations from this chapter. The problems following the equations will help you learn to translate words into equations and to solve single concept problems.

Equations

$\rho = m/V$ (8.1)

$P = \lim_{\Delta A \to 0} (\Delta F/\Delta A)$ (8.2)

$P = \rho g h$ (8.4)

$P_T = P_0 + \rho g h$ (8.5)

$F = f\,A/a$ (8.6)

specific gravity of the solid = $W\text{a}/(W\text{a} - W\text{w})$ (8.7)

$v'A = va$ (8.8)

$P + \rho g h + (½)\,\rho v^2$ = constant (8.15)

$v = [(P_1 - P_2) / 4\eta L]\,(R^2 - r^2)$ (8.17)

rate of flow = $\pi/8\eta\,[(P_1 - P_2)/L]\,R^4$ (8.19)

Problems

1. Water moves through a horizontal pipe (radius = 3 cm) with a velocity of 1 m/sec. Find the speed of the water in a section of pipe that has been constricted to a radius of 1.5 cm.
2. Find the pressure difference between the surface and the bottom of a lake 20.0 m deep. The density of water is 1.00×10^3 kg/m^3.
3. Water is flowing through a horizontal pipe. The velocity of the flow at point A is 1.0 m/sec, and at point B the velocity is 2.0 m/sec. Find the pressure difference between points A and B.
4. Find the flow rate of olive oil through a pipe 0.5 m long with a radius of 1 cm. The viscosity of the oil is 0.18 N-sec/m^2. The pressure difference across the pipe is 2×10^4 N/m^2.
5. A cylinder "weighs" 27 g in air and 17 g in water. Find the specific gravity of the cylinder material.
6. Find the volume of the cylinder in Problem 5.

Answers

1. 4 m/sec
2. 1.96×10^5 N/m^2
3. 1.5×10^3 N/m^2
4. 9×10^{-4} m^3
5. 2.7
6. 10 cm^3

EXERCISES

Section 8.3

1. If the earth's atmosphere were uniform with a density of .00129 g/cm^3, how high would the atmosphere extend? That is, what is the height of a column of air 1 m^2 in area which has a weight of 1.01×10^5 N?[7.99 km]

Section 8.5

2. Suppose that an airplane is flying at an altitude where the atmospheric pressure is 4.03×10^4 N/m^2 and the airplane has gradually become depressurized, so that the internal pressure is equal to atmospheric pressure. The airplane rapidly descends to an airport where the atmospheric pressure is 1.01×10^5 N/m^2. What is the increased force on the eardrum (area 0.300 cm^2) of a passenger? Perhaps you have experienced this sensation to some degree in airplane travel. [1.82 N]
3. A pair of Magdeburg hemispheres (see Figure 8.21) are 10.0 cm in diameter. The barometric pressure is 75 cm Hg. What force will be required to separate them if the interior is completely evacuated? If the pressure inside is 10 cm Hg? [785 N; 680 N]

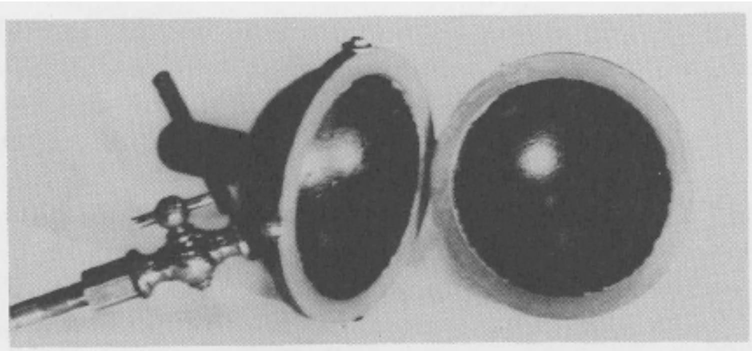


FIGURE 8.21
Magdeburg hemispheres. Exercise 3.

Section 8.6

4. Compute the equivalent pressure of 76.0 cm of Hg in dynes/cm^2 and in newtons/m2. [1.02 x 10^6 dynes/cm^2, 1.02 x 10^5 N/m^2]

5. The systolic blood pressure of an individual is 140 mm Hg, and the diastolic blood pressure of the same individual is 90.0 mm Hg. If the aneroid sphygmomanometer is to be calibrated in N/cm^2, what will the equivalent readings be? For constant volume of blood flow compare the work done by this individual's heart with the work done by the heart of an individual who has high blood pressure of 180 mm Hg systolic and 120 mm Hg diastolic. [1.87 N/cm^2, 1.20 N/cm^2, about 1.3]

6. The U-tube shown in Figure 8.22 contains two liquids that do not interact chemically. The density of one is ρ_1 and the density of the second is ρ_2. What is the ratio of height h_1 and h_2? Suppose one liquid is Hg and the other is oil of specific gravity 0.800. If h_{Hg} is 2.00 cm, what is the height of the oil column? If you wished to construct an open tube barometer to indicate small pressure differences would you use a liquid of high or low density? Why? [$h_1/h_2 = \rho_2/\rho_1$, 34.0 cm]

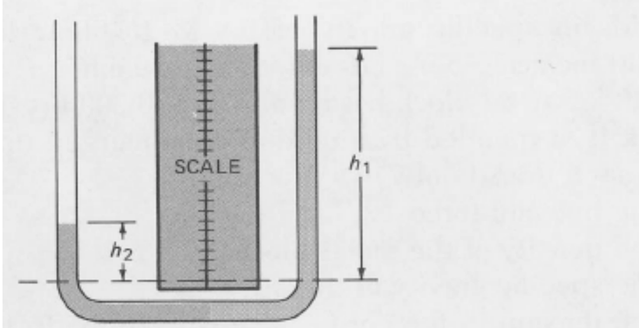


FIGURE 8.22
A U-tube containing two noninteracting liquids.

Section 8.7

7. The gauge pressure in an automobile tire is 2.07 x 10^5 N/m^2. If the wheel supports 4500 N, what is the area of the tire in contact with the road? [2.20 x 10^{-2} m^2]

Section 8.8

8. The areas of the small and large pistons in a hydraulic press are 1.00 cm^2 and 30.0 cm^2. What force must be applied to the small piston in order to lift a 3600-N load? Through what distance must the applied force act if the load is raised two meters? [120 N, 60 m]

9. The piston under a barber's chair is 4 cm in diameter. If the weight of the chair and its occupant is 250 N, how much pressure is required to raise the chair? If this pressure is produced by means of a plunger 0.500 cm^2 in area, what force must be applied to the plunger? [19.9 N/cm^2, 9.95 N]

Section 8.9

10. A uniform stick of wood, 100 cm long and of density 0.70 g/cm^3, is made to float vertically in water. What length is submerged? How deep would it float in a liquid of density 0.80 g/cm^3? Such a stick could be made into a crude hydrometer. If it were to be used in liquids in specific gravity of 0.8 to 1.2, how long would the scale be? [70 cm, 87 cm, 29 cm]

11. A 240-kg metal block has a volume of 0.200 m^3. The block is suspended by a cord and submerged in oil that has a density of 770 kg/m^3. Find
 a. the buoyant force
 b. the density of the metal block
 c. the specific gravity of the oil
 d. the tension in the cord
 [a. 1.51×10^3 N; b. 1.20×10^3 kg/m^3; c. 0.77; d. 840 N]

12. A casting is made of material that has a density of 7.5 g/cm^3. In air the casting weighs 1.47 N. When submerged in the water the casting weighs 1.07 N. Is the casting solid, or does it have a cavity? If it has a cavity, what is the volume of the cavity? If there is no cavity, how much should the casting weigh in water? [hollow, 20 cm^3, 1.27 N]

13. A cylinder of aluminum is weighed on an equal arm balance. The aluminum is balanced by a 200.0-g brass weight. Assume the density of the brass weight to be 8.00 g/cm^3, the density of air to be 1.23×10^{-3} g/cm^3, and the density of aluminum to be 2.7 g/cm^3. What is the buoyant force on the brass weights? What is the buoyant force on the aluminum cylinder? What is the true mass of the aluminum cylinder? [3.02×10^{-4} N, 8.93×10^{-4} N, $(200 + 6.03 \times 10^{-2})$ g]

Section 8.11

14. Assume that from the human aorta, radius of 1 cm, carrying blood from the heart, the cardiac outflow is 5 liters per minute, and the mean blood pressure is 100 mm Hg. Find the velocity of flow in the aorta and the work done per minute. [26.5 cm/sec, 66 J/min]

15. In a perfume aspirator air is blown across the top of the tube which dips into the perfume. What is the minimum air velocity that will cause the perfume to rise to the top of the tube which is 10 cm long if $\rho_{air} = 1.25 \times 10^{-3}$ g/cm^3 and $\rho_{perfume} = 0.9$ g/cm^3. [37.5 m/sec]

16. A circular hole 2 cm in diameter is cut in the side of a large vertical pipe 8 m below the water level in the pipe. What is the velocity of outflow and the volume discharged in one minute? [12.5 m/sec, 0.236 m^3]

17. It is desired to refuel a plane at the rate of 200 liter/min. The fuel line is an 8.00 cm diameter hose connected to a pump 1.00 m above the ground. A 5.00 cm diameter nozzle delivers gasoline to the plane 5.00 m above the ground. Find the speed of the fuel at the nozzle, the speed of the fuel in the line near the pump, and the pressure difference between the pump and the nozzle. (Take the specific gravity of gasoline to be 0.700.) [170 cm/sec, 66.3 cm/sec., 2.83×10^4 N/m^2]
18. A fluid of density 0.800 g/cm^3 is flowing through a 200-cm length of tube that is 1.00 cm in diameter. The flow is found to be 10.0 cm^3/sec, and the pressure drop over the length of the tube is 3.25×10^5 dynes/cm^2. What is the viscosity of the fluid. What are the units? [3.99 poise]
19. Given a hypodermic syringe with an inside diameter of 1.00 cm, and a needle with an inside diameter of 0.500 mm, find the force needed to keep the fluid from coming out of the needle if a 1.00 N force is applied to the plunger. Find the necessary plunger force needed to inject fluid into an artery where the blood pressure is 90.0 mm Hg. [2.50×10^{-3} N, 9.42 N]

Section 8.12

20. Find the pressure necessary to move serum through an intravenous injection tube (radius = 1.00 mm, length = 3.00 cm) at the rate of 1.00 cm3/sec into an artery where the pressure is 100 mm Hg. The viscosity of the serum is 7.00×10^{-3} poise. [1.34 N/cm^2]

PROBLEMS

These problems may involve more than one physical concept. The numerical answer is given at the end of each problem.

21. An airplane weighing 5000 N has a wing area of 12 m^2. What difference of pressure on the two sides of the wing is required to sustain the plane in level flight? Assume the density ρ of air is 1.2 kg/m^3. If the velocity of air relative to the wing is 40 m/sec on the lower side, what is the relative velocity on top of the wing? [420 N/m^2, 48 m/sec]
22. The large section of the pipe in Figure 8.23 has a cross section of 40.0 cm^2 and the small pipe's cross section is 10.0 cm^2. A volume of 30 liters of water is discharged in 5 sec. Find the velocities in both the small and the large pipe. What is the difference in pressure between these portions? What difference in height of a mercury column corresponds to this pressure difference? [150 cm/sec, 600 cm/sec, 1.70×10^5 dynes/cm^2, 127 mm]

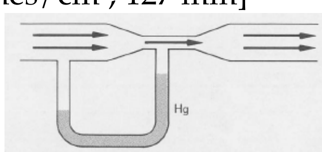


FIGURE 8.23
Liquid flowing in a smoothly tapered tube with two regions of different cross-sectional areas. A U-tube manometer is connected to two parts of the system of different cross-sectional areas. The levels of the mercury in the tube indicate that pressure is lower at the region of higher velocity.

23. Reynolds discovered that the transition from streamline flow (or laminar flow) to turbulent flow (eddy currents form) occurs at a critical relationship among flow velocity, radius of channel, density and viscosity of the fluid. He defined the relationship in terms of the dimensionless number that is now called the Reynolds number *Re:*

    $$Re = (v\rho r)/\eta$$

    where v = flow velocity, ρ = density, r = radius, and η = viscosity. If the value of *Re* is greater than 1000, turbulent flow begins; if *Re* is less than 1000, laminar, or streamline, flow results. When blood undergoes turbulent flow, vibrations are set up in the blood vessels. Assume that the radius of a blood vessel is 0.800 cm, blood viscosity is 0.0200 poise, and the density of blood is 1.00 g/cm^3. Find the velocity that corresponds to the onset of blood vessel wall vibrations [25.0 cm/sec]

24. One method of measuring the viscosity of a fluid is to measure the terminal velocity of a sphere falling through the fluid. The viscous drag force on a sphere is given by Stokes' law:

    $$F = 6\pi\eta r v$$

    where η = viscosity, r = radius of sphere, v = terminal velocity. Show that the viscosity can be expressed as follows:

    $$\eta = (\rho - \rho')\, 2r^2/9v$$

    where ρ = density of sphere, ρ'= density of fluid.

25. The average blood flow velocity in arteries is found to be about 10.0 cm/sec and the average pressure is around 100 mm Hg. Using a density of 1.00 g/cc for blood, compare the energy density due to velocity to that due to pressure. (*Hint:* units of energy density are N/cm^2). If a girl is 165 cm tall and has a blood pressure of 100 mm at her heart, find the blood pressure in her feet 100 cm below her heart. [2.32 N/cm^2]

26. Use the values of blood pressure given in Section 8.13. Assume that each portion of the blood vessel system has a length of 0.8 m, and take the quantity of blood flow to be equal to 1.00 x 10^{-4} m^3/sec. Use the value of 4.00 x 10^{-3} N-sec/m^2 for the viscosity of whole blood. Calculate:

    a. the pressure gradients across each element of the blood vessel system, i.e., aorta, arteries, arterioles, capillaries, and veins

    b. the effective radius for each portion of the system

    c. the average velocity of blood flow through each portion of the system

    [a. 4.99 x 10^2 N/m^3, 2.83 x 10^3 N/m^3, 9.14 x 10^3 N/m^3, 3.33 x 10^3 N/m^3, 1.66 x 10^3 N/m^3; b. 2.13 x 10^{-2} m, 1.38 x 10^{-2} m, 1.02 x 10^{-2} m, 1.33 x 10^{-2} m, 1.58 x 10^{-2} m; c. 7.02 x 10^{-2} m/sec, 1.67 x 10^{-2} m/sec, 3.00 x 10^{-2} m/sec, 1.79 x 10^{-2} m/sec, 1.27 x 10^{-2} m/sec]

27. Assume that the density of your blood is the same as that of water, that your heart is two-thirds of the way up from your feet to your head, that you are 2.00 m tall, and that your average blood pressure is 1.33 x 10^4 N/m2. In an upright position what is the pressure difference between your head and your feet, between your heart and your feet, and between your heart and your head? [1.96 x 10^4 N/m^2, 1.31 x 10^4 N/m^2, 6.50 x 10^3 N/m^2]