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HEMODYNAMICS FOR MEDICAL STUDENTS

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The flow of blood through the cardiovascular system depends on basic principles of liquid flow in tubes elucidated by Bernoulli and Poiseuille. The elementary equations are described involving pressures related to velocity, acceleration/deceleration, gravity, and viscous resistance to flow (Bernoulli-Poiseuille equation). The roles of vascular diameter and number of branches are emphasized. In the closed vascular system, the importance of gravity is deemphasized, and the occurrence of turbulence in large vessels is pointed out.

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Key words: streamline flow; volume flow; velocity; viscous flow pressure; kinetic pressure; gravitational pressure and potential; vascular diameter; number of branches; turbulence

Hemodynamics is the study of the relationship among pressure, viscous resistance to flow, and the volume flow rate ($\dot{Q}$) in the cardiovascular system. Confusion and erroneous thinking have prevailed because of the failure to distinguish the various sources of pressure that may exist and their roles in driving the blood in the "closed" cardiovascular system. Of particular importance is the influence of gravitational pressure in causing or hindering flow. Gravity can induce flow in an "open" system (e.g., from an open container filled with liquid), but it does not cause liquid to flow in a closed system such as the circulatory system (3).

The basic elementary equations related to the flow of fluids are associated with two names, Daniel Bernoulli (Fig. 1) and J. L. M. Poiseuille (Fig. 2). Bernoulli considered, for simplicity, fluids that are nonviscous or frictionless (so-called "ideal") in which pressure may originate from two possible sources: pressure due to the action of gravity on the fluid column (weight of fluid) and/or the pressure related to changes in the velocity of the fluid (inertial forces). The former is commonly called "hydrostatic" pressure, which is often used indiscriminately to describe other sources of pressure as well, such as pressure related to viscous resistance to flow. We will define the pressure due specifically to gravity as "gravitational" pressure, in contrast to other sources of pressure. The pressure to increase velocity is described as "accelerative" pressure, and the pressure resulting from decrease in velocity as "decelerative" pressure.

The Bernoulli equation is often written as follows:

$$\underset{\substack{\swarrow \qquad\qquad \searrow \\ \text{Gravitational} \qquad \text{Accelerative-} \\ \rho\mathbf{gh} \qquad\qquad \text{decelerative}}}{\overset{\substack{\text{(lateral} \\ \text{pressure)}}}{\mathbf{P}}} + \overset{\substack{\text{(gravitational} \\ \text{potential)}}}{\rho\mathbf{gh}} + \overset{\substack{\text{(kinetic} \\ \text{pressure)}}}{\rho\frac{\bar{\mathbf{v}}^2}{2}} = \text{constant} \qquad (1)$$

FIG. 1.
(Reproduced from HYDRODYNAMICS by Daniel Bernoulli and HYDRAULICS by Johann Bernoulli. Translated by T. Carmody & H. Kobus from Latin. New York: Dover, 1968).

The sum of the three components of the equation remains constant in a *given nonviscous* flow system. **P** is the accelerative or decelerative pressure of fluid if there is a change in velocity and/or is due to gravity if there is an elevation from a given reference plane. $\boldsymbol{\rho}$ is the density of the fluid, **g** is the gravitational acceleration (9.8 m/s^2), **h** is the height of the fluid from a given reference plane, $\bar{\mathbf{v}}$ is the mean velocity of the fluid in a flowtube system ($\bar{\mathbf{v}}$ = volume $\dot{\mathbf{Q}}$/cross-sectional area of tube).

The second item (**ρgh**) refers to the gravitational *potential* related to the position or elevation of the fluid from the given reference. The same is true for a solid mass above or below a reference plane.

The third term ($\rho\bar{\mathbf{v}}^2/2$) is the pressure due to the motion of the fluid. It is called "kinetic" or "dynamic" pressure. It cannot be recorded unless the fluid is stopped or decelerated (hence it is also called "impact" pressure). This pressure or force is most obvious in solids. For instance, to move a stopped car, a great force is needed or stopping a fast moving car will damage the car and the object involved in stopping by converting kinetic energy to force (or pressure). In the Bernoulli equation, all three items are interconvertible. Acceleration of fluid converts **P** (lateral pressure) to $\rho\bar{\mathbf{v}}^2/2$; hence **P** falls. This is the well-known Bernoulli principle in which increasing velocity of fluid at a region drops the pressure at that region. It finds many applications in everyday life, e.g., lift of an airplane, garden sprays, flowmeters, etc. Conversely, deceleration converts $\rho\bar{\mathbf{v}}^2/2$ to lateral pressure. Figure 3 illustrates these conversions. It should be remembered that Bernoulli's equation applies to flow that is steady (not pulsatile) and laminar (or streamline) in which the incompressible fluid moves as a series of layers. When flow is fully developed, the molecules immediately adjacent to the wall do not move at all and those in the center move fastest. The velocity profile is parabolic. The average

FIG. 2.
J. L. M. Poiseuille (1799–1869; reproduced, with permission, from Fishman AP and Richards DW. *Circulation of Blood: Men and Ideas*. New York: Oxford University Press, 1964).

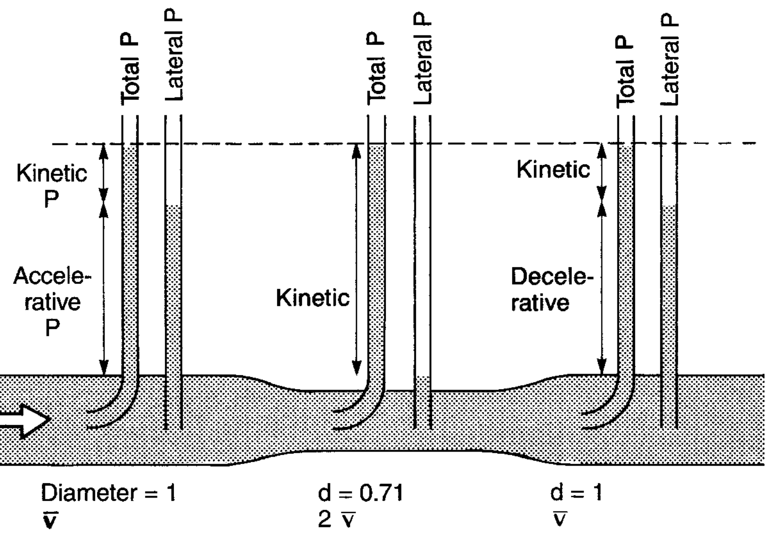


FIG. 3.

Model showing a rigid tube with a narrow segment in the middle to illustrate the conversion of accelerative-to-kinetic and kinetic-to-decelerative pressure (P) in a steadily flowing *nonviscous* liquid (no frictional heat). However, the principle applies equally well to viscous fluids. Thus, when velocity increases, lateral (side) P drops and vice versa (Bernoulli's principle). $\bar{v}$, Mean velocity.

velocity ($\bar{v}$) is equal to one-half the maximum velocity at the center of the circular tube.

The main shortcoming of the Bernoulli equation is that it overlooks the viscosity of fluids. This was studied by a French physician named Poiseuille and a German named Hagen. With the use of glass capillary tubes (rigid) of uniform diameter and a homogeneous liquid (water) with streamline flow, Poiseuille found that the pressure gradient between two points ($P_1 - P_2$) varies.

1. Directly with the distance between P_1 and P_2 (**L**). Therefore,
   $P_1 - P_2 \propto L$ (constant $\dot{Q}$, **r**, and η)
   where **r** is internal radius and η is viscosity of liquid (Fig. 4*A*).

2. Directly with the flow rate, $\dot{Q}$ (Fig. 4*B*).
   $P_1 - P_2 \propto \dot{Q}$ (constant **L**, **r**, η)

3. Directly with the η (Fig. 4*C*)
   $P_1 - P_2 \propto \eta$ (constant **L**, $\dot{Q}$, **r**)

4. Inversely with the 4th power of radius (Fig. 4*D*)
   $P_1 - P_2 \propto \frac{1}{r^4}$ (constant **L**, $\dot{Q}$, η)
   Therefore, $P_1 - P_2 \propto L\eta\frac{\dot{Q}}{r^4} = k\,L\eta\frac{\dot{Q}}{r^4}$
   k was found to be $\frac{8}{\pi}$

So

$$P_1 - P_2 = \frac{8}{\pi}\frac{L\eta\dot{Q}}{r^4} \quad \text{or} \quad \dot{Q} = \frac{(P_1 - P_2)\pi r^4}{8L\eta} \qquad (2)$$

This relationship is known as *Poiseuille's equation* or the *Hagen-Poiseuille equation*. It can be derived also mathematically. It should be emphasized that $P_1 - P_2$ in the Poiseuille equation is *strictly limited to the viscous loss of pressure in a flowtube, excluding the pressures shown in the Bernoulli equation*. In other words, *it excludes the difference in the gravitational or accelerative-decelerative* pressures that may exist between the two points in a tube system. To emphasize its importance, we refer to this pressure as the

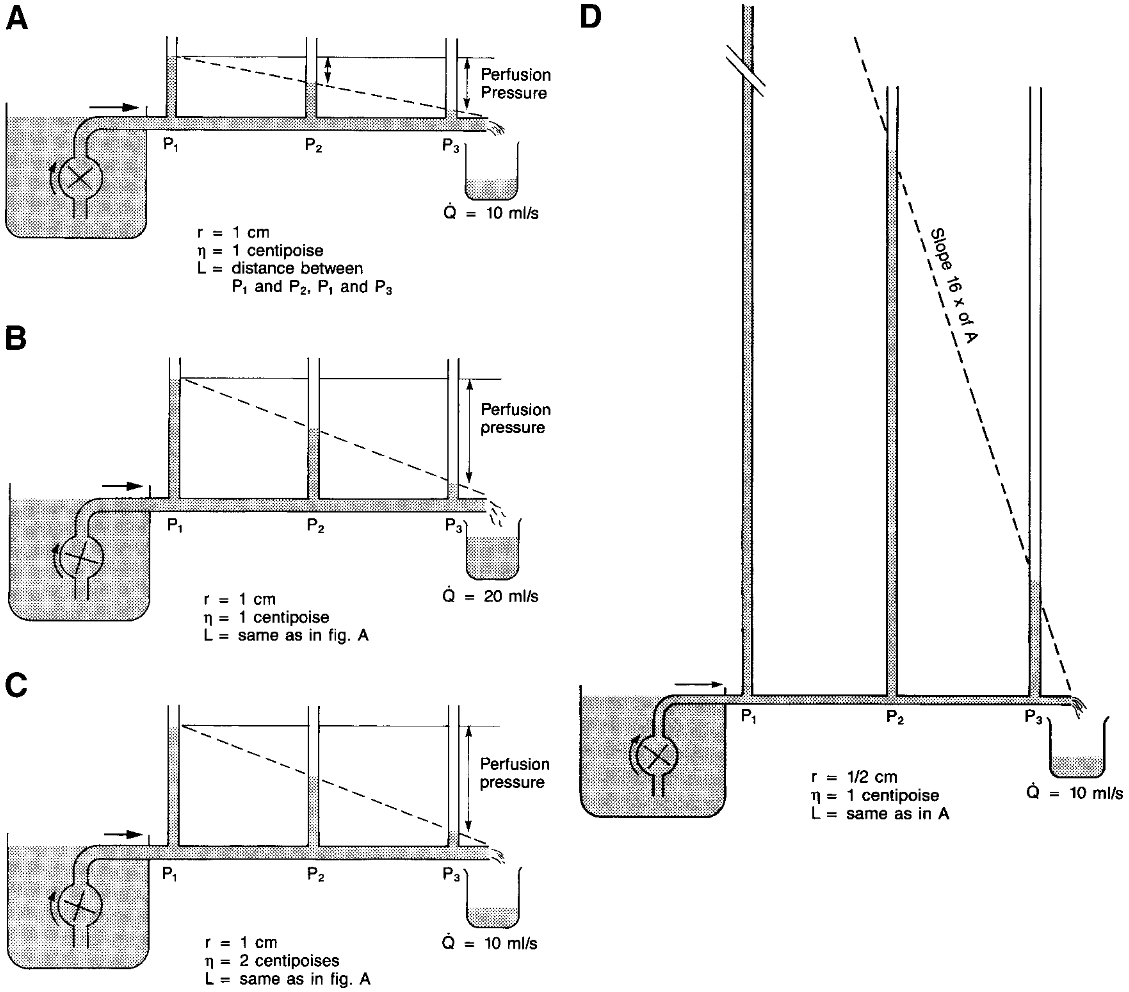


FIG. 4.

***A*: pumping a homogeneous *viscous* liquid at a steady rate through a rigid tube of uniform diameter. The lateral Ps drop along the tube, indicating the viscous flow P gradients. The perfusion Ps between the different points vary directly with the distance between the points (L). Drop of P is due to friction (heat loss). *B*: keeping L, viscosity (η) and radius (*r*) constant, and doubling the flow rate ($\dot{Q}$) doubles the perfusion P ($P_1 - P_2$). *C*: keeping $\dot{Q}$, L, and *r* as in *A*, and doubling the η doubles ($P_1 - P_2$). *D*: effect of diameter on viscous flow P gradient. Reducing the diameter to one-half increases the pressure gradient 16 times, other things being as in *A*.**

viscous flow pressure (1). Viscous flow pressure drops continuously along a flowtube because of *frictional heat between the particles of the fluid*. This heat cannot be reconverted into pressure (considered "lost").

In a single rigid tube of uniform diameter with the steady flow of a homogeneous fluid, one can calculate the $\dot{\mathbf{Q}}$ if $\mathbf{P_1} - \mathbf{P_2}$, **L**, **r**, and η are known.

Viscous resistance to flow. It is customary to refer to $8L\eta/\pi r^4$ as the resistance (**R**) to flow between two points in a single flowtube. Hence, the Poiseuille equation may be generalized as $\mathbf{P_1} - \mathbf{P_2} = \mathbf{R}\dot{\mathbf{Q}}$ or $\mathbf{R} = \mathbf{P_1} - \mathbf{P_2}/\dot{\mathbf{Q}}$. If $\dot{\mathbf{Q}}$ is constant, $\mathbf{P_1} - \mathbf{P_2}$ may be taken to indicate **R**. This equation is analogous to Ohm's law in electrical circuits, **E = IR** (volts = amperes × ohms) or **I = E/R**. The general equation is applicable also when tubes divide and subdivide and then reunite, as

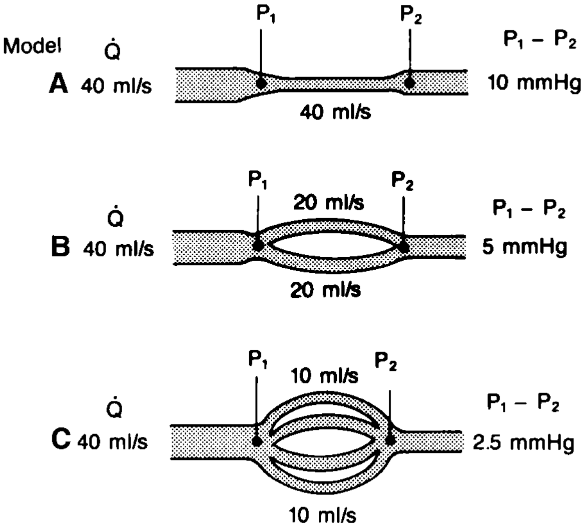


FIG. 5.

Other things being equal ($\dot{Q}$, diameter, L, and η), viscous flow pressure gradients vary *inversely* with the number of "parallel" tubes (reproduced, with permission, from H. S. Badeer and D. H. Petzel. *Am J Nephrol* 15: 95, 1995).

it is in the circulatory system. The branches are tubes that are placed "in parallel," and the total **R** ($\mathbf{R_T}$) of such a flow system is similar to the equation in electrical circuits

$$\frac{1}{\mathbf{R_T}} = \frac{1}{\mathbf{R_1}} + \frac{1}{\mathbf{R_2}} + \frac{1}{\mathbf{R_3}} \tag{3}$$

and so on.

If flow is kept constant, the more the branches, the less is the $\mathbf{R_T}$ or the pressure drop (Fig. 5). Another consequence of "parallel" tubes is that the $\mathbf{R_T}$ is less than any of the individual **R**. For example, if the resistances of the four parallel elements in Fig. 5*C* were equal, then $\mathbf{R_1} = \mathbf{R_2} = \mathbf{R_3} = \mathbf{R_4}$. Therefore, $1/\mathbf{R_T} = 4/\mathbf{R_1}$ and $\mathbf{R_T} = \mathbf{R_1}/4$.

The influence of number of vessels is an important principle to remember in the circulatory system. A good example is the very small pressure drop in the renal glomerular capillary network that consists of 20–40 capillaries "in parallel." Figure 6 represents the pressure drop in the systemic vascular circuit. It is seen that the arterioles are the chief resistant vessels,

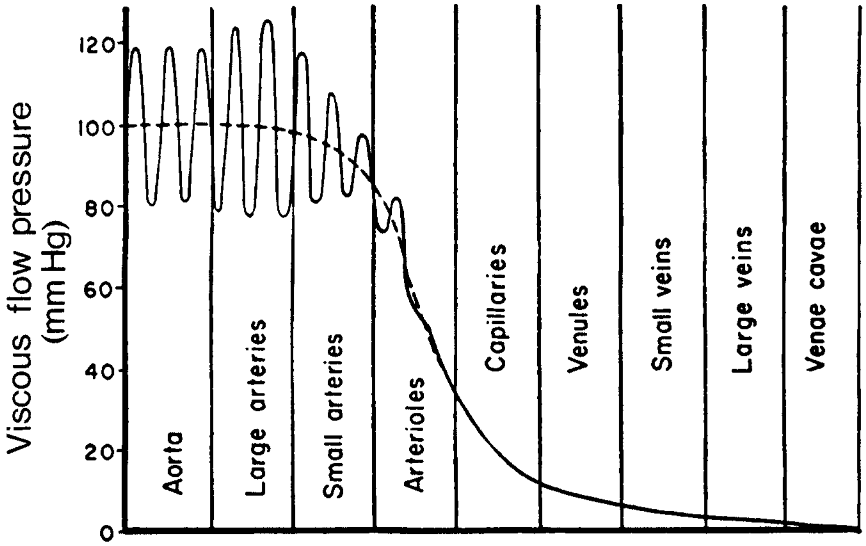


FIG. 6.

Viscous flow pressures in the systemic vascular circuit. Note, the greatest drop of P ($P_1 - P_2$) is in the arterioles, which represent the chief resistance vessels. Next largest drop is in the capillaries. There is very little viscous resistance in the large arteries and veins. [Reproduced, with permission, from A. C. Guyton. *Textbook of Medical Physiology* (7th ed.). Philadelphia PA: Saunders, 1986).

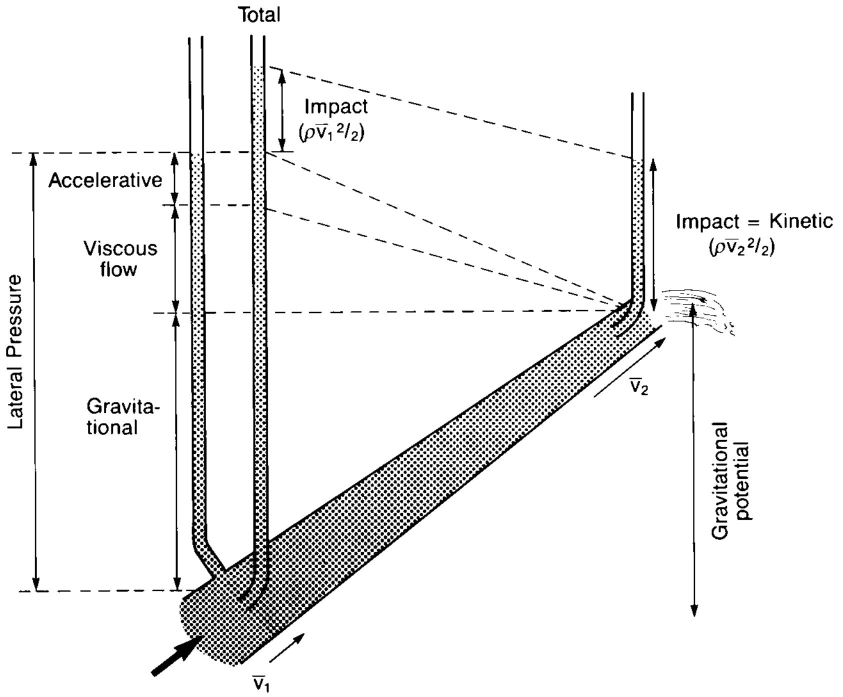


FIG. 7.

Model illustrating all the components of the Bernoulli-Poiseuille equation. (Reproduced, with permission, from H. S. Badeer. *The Physics Teacher* 32: 426, 1994).

which is explained by the smaller number and greater effective viscosity of blood in arterioles compared with that of capillaries. Whereas in a single tube with homogeneous liquid flow, it is possible to calculate resistance by applying the equation $8\mathbf{L}\eta/\pi\mathbf{r}^4$, in a complex vascular network, it is impossible to use this equation. One has to resort to the general equation, $\mathbf{R} = \mathbf{P}_1 - \mathbf{P}_2/\dot{\mathbf{Q}}$. If $\dot{\mathbf{Q}}$ is constant, $\mathbf{P}_1 - \mathbf{P}_2$ indicates the **R** (in mmHg · ml^{-1} · min^{-1} or peripheral resistance units) of the system.

If resistances are placed "in series," the $\mathbf{R}_\mathbf{T}$ is equal to the sum of the individual **R**s. Some authors prefer to use the reciprocal of **R**, which is designated as *conductance*. Conductance = 1/**R**.

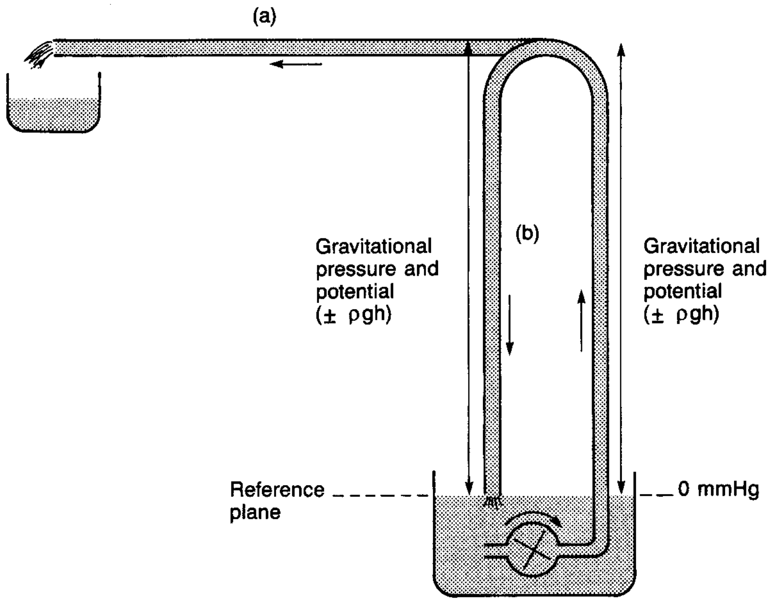
Bernoulli-Poiseuille equation. It was pointed out earlier that Bernoulli's equation is incomplete because it overlooks the viscosity of fluids (η). Likewise, Poiseuille's equation is incomplete because it neglects gravitational pressure and gravitational potential as well as accelerative-decelerative pressures. The more realistic equation is a combination of the two, which has been designated as the *Bernoulli-Poiseuille equation* (1).

$$\underset{\substack{\diagup\quad|\quad\diagdown\\ \text{Gravitational}\quad|\quad\text{Accelerative}\\ \text{Viscous flow}}}{\mathbf{P}} + \rho\mathbf{gh} + \frac{\rho\bar{\mathbf{v}}^2}{2} \qquad (4)$$

$$+ \int \text{Viscous losses} = \text{a constant}$$

Figure 7 illustrates all of the components of this equation in a graphic way. Flow of liquids is due to the gradients in total pressure or energy (**PΔV**; excluding thermal) that takes into account the various sources of pressure or energy.

In the circulatory system, the influence of kinetic, compared with viscous flow pressure, is relatively



Position (a) Work of pump = Gravit. P (ρgh) + Viscous P ($\dot{Q}R$) + Gravit. potential (ρgh) + Kinetic P ($\rho\bar{v}^2/2$)

Position (b) Work of pump = Viscous P ($\dot{Q}R$) + Kinetic P ($\rho\bar{v}^2/2$)

FIG. 8.

Model showing the reduction of work of pump in an inverted U tube system. In position (a), the pump develops P to overcome the gravitational P (ρgh) in addition to viscous P and kinetic P. In position (b), the gravitational effects in the upward limb are counterbalanced by the gravitational effects in the downward limb of the loop, leaving only the viscous P gradient ($\dot{Q}R$) and the kinetic pressure [$\rho(\bar{v}^2/2)$].

small because of its magnitude. It is significant in a few areas of the circulatory system. For instance, it plays a role in driving blood from the left ventricle into the aorta during the latter part of the period of ejection (see cardiac cycle). Also, it plays a role in the pulmonary circuit and the filling of the ventricles with blood from the atria (2). In other parts of the circulation, the contribution of velocity of flow (hence kinetic pressure) in driving blood is negligible.

Although normally in the circulatory system, the accelerative-decelerative and kinetic components of pressure are not very significant, gravitational pressure and gravitational potential of blood become significant in the upright position of man and animals. If a flowtube with an *open aperture* is directed upward or downward (open system), gravity hinders upward flow or facilitates downward flow. If the liquid is pumped upward and is discharged to a higher gravitational potential, the pump develops pressure to overcome gravitational pressure (**ρgh**) and to overcome the viscous resistance of the tube (**$\dot{Q}$R**). It lifts the liquid to a higher gravitational potential (**ρgh**; Fig. 8*A*).

If now the same tube is bent down as shown in Fig. 8*B*, the liquid is returned to its original level of gravitational potential. This design eliminates the work of the pump to overcome gravitational effects (pressure and potential), because the gravitational effects in the downward limb counterbalance the gravitational effects in the upward limb of the circuit (counterbalancing principle; Fig. 9). The pump now develops pressure to *overcome the viscous resistance of the*

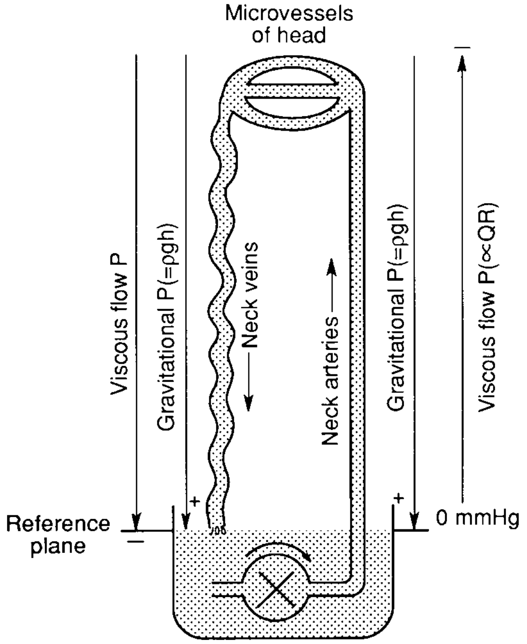


FIG. 9.

Model showing the counterbalancing of the ρgh in the neck arteries by the equal and opposite ρgh in the neck veins. Arrow with a plus (+) sign, the direction of increasing P; arrow with a minus (−) sign, the direction of decreasing pressure. (Reproduced, with permission, from H. S. Badeer. *Comp Biochem Physiol* 118A: 575, 1997).

tube and impart velocity. This may be considered to be a "closed" system.

A similar situation exists in the circulatory system, where blood is pumped out of the ventricles and is returned to the ventricles by way of the veins and atria in a U or inverted U design below and above the heart (closed system). In other words, *gravity neither favors nor hinders blood flow in the upright position*. This important principle in hemodynamics is often overlooked or misunderstood. To illustrate, let us say that in the upright position of a person, the mean pressure in the right atrium is 2 mmHg and the mean pressure in the dorsal vein of the foot is 90 mmHg. Most of the 90 mmHg is due to the pressure of the column of blood from the foot to the right atrium (**ρgh**). If this gravitational pressure is ~85 mmHg, then the *driving or perfusion* pressure from the foot to the right atrium is not 90–2 mmHg, but is (90–85)-2 = 3 mmHg. In other words, *perfusion pressure excludes the difference in gravitational pressure between two points and refers only to the viscous flow pressure gradient*. Although the model in Fig. 8 refers to a rigid tube system, in a collapsible tube system such as the circulatory, the same principle applies except that in the upright position, gravitational pressure alters the diameter of blood vessels above and below the heart. Gravitational pressure drops above the heart and increases below the heart. Accordingly, vessels above the heart tend to collapse, and vessels below the heart distend. The veins being more compliant are affected most (compliance is the ratio of change of volume to change of pressure). Venous return from the lower parts of the body tends to be reduced and causes a drop of cardiac output and arterial pressure. There are important protective mechanisms to maintain arterial pressure and blood flow to the head in the upright position (see carotid sinus reflex).

Students who are interested in experimenting may use the model described by Smith (5).

Distinction between volume $\dot{Q}$ and velocity of flow. It is important to distinguish between *volume* $\dot{\mathbf{Q}}$ and *velocity* of flow. $\dot{\mathbf{Q}}$ is volume per unit time, whereas velocity is distance per unit time. By definition, $\bar{\mathbf{v}} = \dot{\mathbf{Q}}$/cross-sectional area, or in a circular tube

$$\bar{\mathbf{v}} = \frac{\dot{\mathbf{Q}}}{\pi \mathbf{r}^2} \qquad (5)$$

If we substitute for $\dot{\mathbf{Q}}$ from the Poiseuille equation we obtain

$$\bar{\mathbf{v}} = (\mathbf{P}_1 - \mathbf{P}_2)\,\frac{\pi \mathbf{r}^4}{8\mathbf{L}\eta} \times \frac{1}{\pi \mathbf{r}^2} = (\mathbf{P}_1 - \mathbf{P}_2)\,\frac{\mathbf{r}^2}{8\mathbf{L}\eta} \qquad (6)$$

Consequently, if $\mathbf{P}_1 - \mathbf{P}_2$, **L**, and $\boldsymbol{\eta}$ are *kept constant*, increasing the radius *increases* the velocity of flow. This may appear paradoxical because increasing **r** increases the cross-sectional area, and this might be expected to decrease the velocity. That would be true if $\dot{\mathbf{Q}}$ were to remain unchanged; but we see from the Poiseuille equation that when **r** increases and $\mathbf{P}_1 - \mathbf{P}_2$ is kept constant, $\dot{\mathbf{Q}}$ would increase very markedly (by $\mathbf{r}^4$). Because flow increases by $\mathbf{r}^4$ and the cross-sectional area increases by $\mathbf{r^2}$, velocity, which is flow divided by cross-sectional area, increases by $\mathbf{r^2(r^4/r^2)}$.

This situation occurs in the body when blood vessels dilate *locally* without causing any change in mean arterial blood pressure. Both blood flow and velocity increase during local vasodilation in a tissue and vice versa.

Turbulent flow. In contrast to streamline flow, turbulent flow occurs when fluid particles move randomly in all directions in a flowtube or vessel. Such flow requires more energy or pressure than streamline flow. Osborne Reynolds (1842–1916) studied this in a model and established that turbulent flow occurs when a number called Reynolds' number exceeds 2,000. It is calculated as follows: Reynolds' number = $\mathbf{d}\bar{\mathbf{v}}$ (ρ/η) (dimensionless), where $\mathbf{d}$ is the internal diameter of tube (cm) and ρ is density.

From this equation, it is seen that large tubes with high velocity of flow tend to develop turbulence. Aortic and pulmonary artery flow during ejection tend to develop turbulence that contributes to the first heart sound.

In disease, turbulence occurs when blood flows through narrow cardiac valves at high velocity or when arteries are narrowed locally (e.g., carotid artery). Vibrations are produced that may be heard with a stethoscope. Turbulence also occurs when blood flows in opposite directions (regurgitation through a cardiac value). These abnormal sounds over the heart are known as murmurs (4).

Turbulent flow occurs when one is taking arterial pressure with the cuff around the arm and stethoscope at the elbow. When cuff pressure is reduced slowly, the jet of blood passing under the cuff with each heart beat flows through the stationary blood below the cuff and causes vibrations of the artery that can be heard with a stethoscope (Korotkoff sounds) (4).

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REFERENCES

1. **Badeer HS and Synolakis CE.** The Bernoulli-Poiseuille equation. *The Physics Teach* 27: 598–601, 1989.
2. **Burton AC.** *Physiology and Biophysics of the Circulation*. (2nd ed.) Chicago: Year Book Medical, 1972, p. 104–106.
3. **Hicks JW and Badeer HS.** Gravity and the circulation: "open" vs. "closed" systems. *Am J Physiol Regulatory Integrative Comp Physiol* 262: R725–R732, 1992.
4. **Rushmer RF.** *Cardiovascular Dynamics* (4th ed.) Philadelphia: Saunders, 1976, p. 184 and 433.
5. **Smith AM.** A model circulatory system for use in undergraduate physiology laboratories. *Am J Physiol Adv Physiol Educ* 22: S92–S99, 1999.