Chapter 8
Fluid Flow

GOALS

When you have mastered the contents of this chapter, you will be able to achieve the following goals:

Definitions

Define each of the following terms, and use it in an operational definition:

- fluid
- density
- specific gravity
- pressure (absolute and gauge)
- buoyant force
- streamline flow
- viscosity

Fluid Laws

State Pascal's law of hydrostatic pressure, Archimedes' principle of buoyancy, Bernoulli's equation for the conservation of energy in a fluid, and the law of conservation of fluid flow.

Fluid Problems

Solve problems making use of the principles of fluids and conservation laws.

Viscous Flow

Use Poiseuille's law of viscous flow to solve numerical problems.

PREREQUISITES

Before you begin this chapter, you should be able to solve problems that use energy concepts (see Chapter 5).

Chapter 8
Fluid Flow

OVERVIEW

The movement of liquid or gaseous substances is an important consideration for medical doctors and air conditioning mechanics, as well as children using a soda straw. There seems to be endless examples of how we are dependent on the flow of fluids for making the work we do easier and more complete.

SUGGESTED STUDY PROCEDURE

To begin your study of this chapter, read the following Chapter Goals: Definitions, Fluid Laws, Fluid Problems, and Viscous Flow. Expanded discussion of each term under Definitions is found in the next section of this Study Guide chapter.

Next, read text sections 8.1-8.13. Remember that answers to the questions asked in these sections are given in this Study Guide chapter. Now read the Chapter Summary and complete Summary Exercises 1-12. Now do Algorithmic Problems 1, 2, 3, and 4, and complete Exercises and Problems 2, 3, 4, 5, 7, 8, 9, 15, 17, and 21. For additional experience, select from the other Exercises and Problems and/or consider the Examples given in the third section of this chapter. Now you should be prepared to attempt the Practice Test on Fluid Flow at the end of this chapter. Remember to wait until you have worked the problem before looking at the answer. Seek assistance in areas where you do not score 100%. This study procedure is outlined below.

Chapter Goals	Suggested Text Readings	Summary Exercises	Algorithmic Problems	Exercises & Problems
Definitions	8.1,8.2,8.3, 8.4	1,2,3,4, 5,6,7		
Fluid Problems	8.5,8.6,8.7, 8.8	8,9,10,11	2	2,3,4,5, 7,8,9,15
Viscous Flow	8.10,8.11,8.12 8.13	12	1,3,4	17,21

DEFINITIONS

FLUID

A substance which takes the shape of its container and flows from one location to another.

The class of all materials that are fluids includes both liquids, such as water, and gases, such as air. The class of incompressible fluids is called liquids.

DENSITY

The ratio of the amount of matter contained in an object to the amount of space occupied by the object is called its density.

In the SI system the density of water at a temperature of 4°C is defined as exactly 1000 kilograms of mass for each cubic meter of water, or 1 gm per cubic centimeter of water.

SPECIFIC GRAVITY

The ratio of the density of a substance to the density of water.

The specific gravity of an object that sinks in water is a number larger than 1.0. Note that specific gravity is a number with no units.

PRESSURE

The normal force acting on a unit area is the pressure.

Since all of the objects near the surface of the earth are usually acted upon by the pressure of the earth's atmosphere of air, we often measure only the excess pressure of a system. We then call it a gauge pressure. We must add the atmospheric pressure to the gauge pressure to obtain the value of the absolute pressure.

BUOYANT FORCE

The resultant of all the pressure forces acting on an object submerged in a fluid.

In general, the buoyant force pushes up on an object making its weight in a fluid less than it would be in a vacuum. The greater the density of the fluid in which you weigh the object the larger is the buoyant force.

STREAMLINE FLOW

Every particle in a flow which passes a given point will flow through all the points in the line of flow.

Such flow has no swirls or turbulence. In general it is only approximated in real fluids by low speed flow in smooth-walled containers.

VISCOSITY

The tendency of a real fluid to resist flow is known as viscosity. It is the internal friction of the fluid. The liquid coefficient of viscosity is a relative measure of liquid friction and is equivalent to the ratio of the force per unit area to the change in velocity per unit length perpendicular to the direction of flow.

The most common example of the importance of viscosity is probably in the motor oils used to lubricate automobile and motorcycle engines. The greater the viscosity of a fluid the greater is its resistance to flow. Cold syrup has a greater resistance to pouring than warm syrup. Viscosity of many liquids is strongly temperature dependent.

ANSWERS TO QUESTIONS FOUND IN THE TEXT

SECTION 8.1 Introduction

The circulation of air in a building to provide proper temperature of the habitation of human beings is an example of using fluid flow to transfer energy from one location to another. The flow inertia of a fluid is measured by its density. The loss of energy in the flow of a liquid is called viscosity.

SECTION 8.3 Density

The dimensions of density are M/L^3. To covert kg/m^3 to gm/cm^3 divide by 1000 or 10^3.

SECTION 8.4 Force on Fluids

If you try to push your hand along the surface of a fluid, parallel to its surface, you force the fluid to travel along with your hand with very little resisting force. As you slid your hand along a solid surface you feel a frictional force opposing the motion of your hand.

SECTION 8.5 Pressure

Because of the persistent use of the English system of units, a common pressure unit is pounds per square inch. The pressure for inflation of bicycle tires is given in lb. per square inch. Tire pressures are intended to be gauge pressures. Pressure is also given in atmospheres, 1 atmosphere of pressure being equal to the pressure caused by the usually atmospheric pressure on the surface of the earth. The weather bureau gives the pressure in inches of mercury; i.e., the height of a column of mercury that is supported by the pressure of the atmosphere.

SECTION 8.6 Pressure of a Liquid in a Column

We can use Equation 8.4 in conjunction with our knowledge of the density of mercury $13.6 \times 10^3 kg/m^3$ and the gravitational constant $9.8\ m/s^2$ to calculate the standard atmospheric pressure in newtons per square meter, $P = (13.6 \times 10^3\ kg/m3)(9.80\ m/s^2)(0.760\ m) = 1.01 \times 10^5\ N/m^2$.

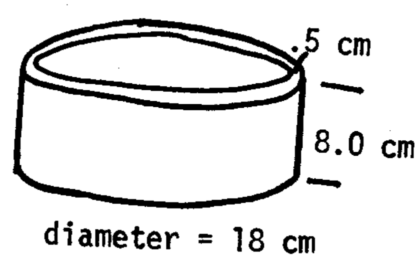
SECTION 8.9 Archimedes Principle

Two ways to measure the density of a liquid are (1) direct measurement of the mass of a known volume of the liquid using a container whose volume has been calibrated, or (2) float an object of known mass and volume in the liquid and measure the portion of the object that remains above the surface of the liquid. In the second method Archimedes Principle is used to calculate the weight of displaced liquid.

To perform some sample calculations from the traditional Archimedes story let us assume the crown was a hollow cylinder of 56 1/2 cm circumference, 0.50 cm in thickness and averaged 8.0 cm in height, then the volume of the crown is approximately

$V \approx 2\pi rh(\Delta t) = 2\pi\ (9\ cm)(8\ cm)(0.5\ cm)$

$V \approx 226\ cm^3$



If the crown were pure gold it would have a mass of 4.4 kg. If the crown were pure lead it would have a mass of 2.6 kg. One possibility is that Archimedes stepped into a completely full bath tub causing the water to overflow. He then realized he could use the same technique by catching the overflowing water to find the volume of the crown. He could then perform a straight-forward comparison of the density of the crown to the density of gold. However, mythology has it that Archimedes was extremely brilliant, so he no doubt worked out the Greek equivalent to Equation (8.7) on the way running to the palace; i.e.

$\Sigma\ \Gamma = A/\ (A\text{-}\Omega)$ (1)

SECTION 8.12 Poiseuille's Law of Viscous Flow

Example 2. The flow of the I.V. liquid will be determined by its viscosity, the diameter of the injection needle and the pressure head, h. If the diameter is reduced by one-half, then by Equation 8.19 the flow is reduced to 1/16th its original value so the pressure head would have to be increased to 16 h!

EXAMPLES

FLUID PROBLEMS

1. The water flowing from a garden hose is passed through a nozzle and then sprayed into the air. Assume the hose is in a horizontal position and the pressure and velocity of the water in the hose are $3.0 \times 10^5\ N/m^2$ and 2.6 m/s respectively. If the nozzle reduces the effective area by 84%; i.e., area of nozzle = (0.16) area of hose, what happens to the water pressure and water velocity in the nozzle?

What Data Are Given?

$P_{hose} = 3.0 \times 10^5\ N/m^2$; $v_{hose} = 2.6\ m/s$; $A_{nozzle} = 0.16\ A_{hose}$

What Data Are Implied?

The height of the hose and the nozzle are the same. The water may be treated as an ideal, streamline flow fluid.

What Physics Principles Are Involved?

We can make use of the conservation of liquid volume (Equation 8.8) and the conservation of energy (Equation 8.14).

What Equations Are to be Used?

$v_h A_h = v_n A_n$ (8.8)

$P_h + pgh_h + (½)\, pv_h^2 = P_n + pgh_n + (½)\, pv_n^2$ (8.14)

where the subscripts h and n stand for hose and nozzle respectively.

Algebraic Solution

We are given that the elevation of the hose and the nozzle are the same, so we can proceed as follows.

The velocity in the nozzle $v_n = (A_h v_h) / A_n$ (2)

The velocity is increased by a factor equal to the ratio of the areas.

We can substitute Equation (2) into equation (8.14) and solve for the pressure in the nozzle; noting that $pgh_h = pgh_n$

$P_n = P_h + (½)\, p(v_h^2 - v_n^2)$

$P_n = P_h + (½)\, pv_h^2\, ((1 - A_h^2) / A_n^2)$ (3)

NOTE! The area of the hose A_h is larger than the area of the nozzle A_n so the second term on the right-hand side of equation (3) is negative. The pressure in the nozzle must be less than the pressure in the hose!

Numerical Solution

$v_n = (A_h/0.16\, A_h)(2.6\ m/s) = 16\ m/s$

$P_{nozzle} = (3.0 \times 10^5) + (½)\, (10^3\ kg/m3)\, (2.6\ m/s)^2\, (1 - (A_h/0.16\, A_h)^2)$

$= 3.0 \times 10^5 + (½)\, (6.8 \times 10^3)(-38)$

$P_n = 3.0 \times 10^5 - 1.3 \times 10^5 = 1.7 \times 10^5\ N/m^2$

Thinking About the Answers

The velocity in the nozzle is increased by more than six times while the nozzle pressure is 57% of the pressure in the hose. Notice that the high velocity region is also the region of low pressure.

VISCOUS FLOW

2. When taken from a refrigerator (4ºC) cold human blood has a viscosity of 8 centipoise. At room temperature (20º) its viscosity is 4 centipoise. Assume the viscosity of blood changes linearly with temperature and that the blood flow was proper for cold blood for an IV arrangement as shown in Figure 8.15 with a bottle elevation of ho. Explain how the elevation of the bottle should be changed as the blood warms up to maintain a constant flow.

What Data Are Given?

The viscosity of blood at 4ºC is 8 centipoise. The initial elevation head for the flow of the blood is h_o.

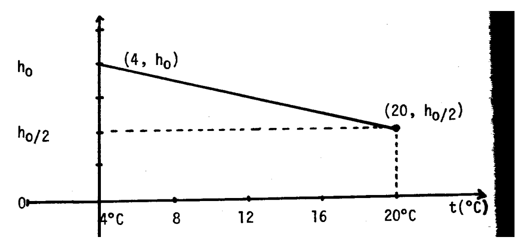
What Data Are Implied?

It is assumed that Poiseuille's Law is applicable. The viscosity of the blood can be related to the temperature by a linear function. Since the viscosity at two temperatures is known and the general form of the equation must be viscosity = $(\text{constant})_1$ temperature + $(\text{constant})_2$ the quantitative relationship between viscosity and temperature can be derived,

$\eta(\text{centipoise}) = -(1/4)\, t(^\circ c) + 9$ (4)

for the temperature region $4 \leq t \leq 20$.

What Physics Principles Are Involved?
The flow of human blood is viscous flow, so Poiseuille's Law may be applied, Equation 8.19.
What Equations Are to be Used?
rate of flow = $(\pi/8\eta)\ (P_1 - P_2)/L)\ R^4$ (8.19)
and Equation (4) $\eta = -t/4 + 9$ (4)
pressure difference = $P_1 - P_2 = pgh$ (8.4)
Algebraic Solution
In order for the rate of flow to be a constant as the viscosity of the blood changes the pressure, or elevation head, must be changed. Since all the other variables except h and h are constant, then n and h must be changed so that they maintain a constant ratio;
rate of flow = $(\pi/8)\ (R^4/L)\ (pgh/\eta)$
At the beginning for cold blood (η = 8 centipoise) the elevation is h_o; so
$h/\eta = h_o/(8$ centipose$) = h/(9 - t/4)$
Solving for h; $h = (h_o/8)\ (9 - t/4) = 9h_o/8 - h_o/32\ t$ (5)
Numerical Solutions
We can draw a graph of the elevation of the bottle as a function of the temperature of the blood.



Thinking About the Answer
As the blood becomes "thinner," its viscosity is diminished and the bottle is lowered to lower the elevation head in order to maintain a constant flow.

3. The flow of a viscous liquid through a tube is constricted by a narrow rubber hose connection. As the connection ages, the rubber expands and the diameter of the constriction increases by 8%. How much does the flow rate increase?
What Data Are Given?
For the normal constriction there is some standard rate of flow. The diameter of the constriction increases from d to 1.08 d.

What Data Are Implied?

It is assumed that the conditions are correct for Poiseuille's Law to apply.

What Physics Principles Are Involved?

The streamline flow of a viscous liquid as described by Poiseville's Law, Equation 8.19, are assumed to apply.

What Equation is to be Used?

rate of flow = $(\pi/8\eta)\ (P_1 - P_2/L)R^4$ (8.19)

Algebraic Solution

All of the variables of the system are constant except for the diameter of the tubing; so

(rate of flow)$_1$ ∞ (radius)$_1^4$ = ((diameter)$_1^4$) / 16

so rate of flow ∞ (diameter)^4

2nd flow rate / 1st flow rate = (second diameter)^4 / (first diameter)^4 (6)

Numerical Solution

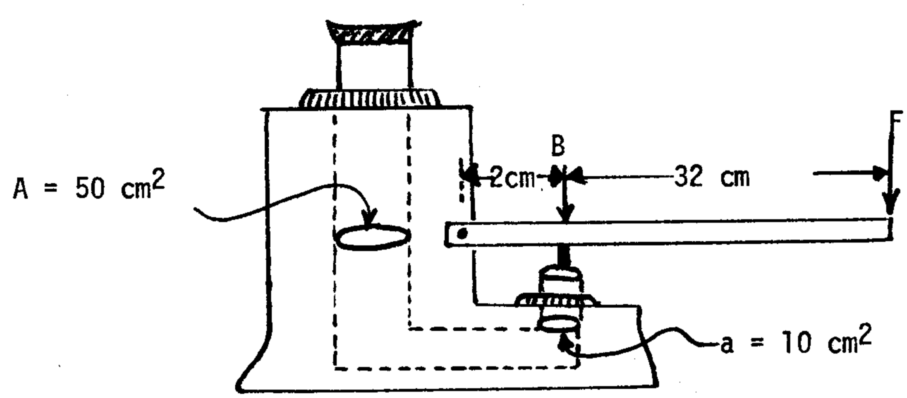
2nd flow rate/1st flow rate = $(1.08\ d)^4/d^4 = (1.08)^4 = 1.36$

Thinking About the Answer

A relatively small increase in the size of the tube (8%) makes a large (36%) increase in the flow rate.

PRACTICE TEST

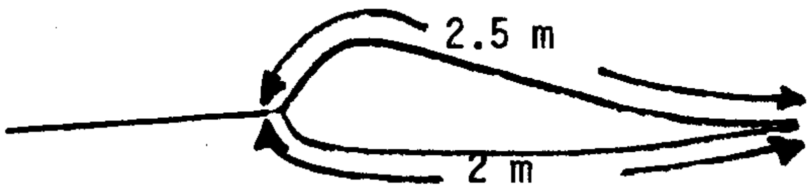
1. A schematic diagram representing a hydraulic jack pictured below is being used to lift the front of a car off the ground. A force of 10,000 Newtons is required.



a. What force at B is required on the small piston to produce the needed force of 10,000 N?

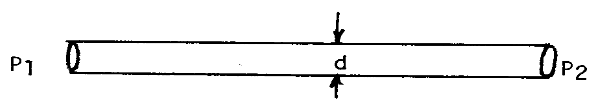
b. Under ideal conditions, what force F is required at the end of the 34 cm lever arm to produce this force?

2. A jet airplane is flying at an elevation of 10 kilometers where the air has a density of 4.25×10^{-1} kg/m^3 and a pressure of 2.7×10^4 N/m^2. The velocity of the air along the bottom of the wing is 20 m/sec. Given the dimension of the wing as shown below:



a. What is the velocity of the air along the top of the wing? (Assume that the air flow along the top and the bottom of the wing requires the same time.)
b. What is the magnitude of the unbalanced pressure acting up on the wing?
c. If the airplane has a total wing area of 150 m2, what weight can the airplane support?

3. A person with hardening of the arteries can survive an effective diameter decrease of his arteries of 20%. Under these conditions, how much must the blood pressure increase to keep the rate of blood flow constant?
Hint: (Assume that the viscosity of blood remains constant and that the blood flow is unchanged and use Poiseuille's Law in a modified form)



ANSWERS:

1. 2,000 N, 118 N
2. 25 m/s, 49 N/m^2, 7,200 N
3. 2.4 times the initial pressure