1. Consider the linear regression model \( Y = X\beta + \varepsilon \) where \( \varepsilon \sim N_n(0, \sigma^2 I_n) \). Let \( H \) be the hat matrix. Define, \( SSR_M = Y'HY \) and \( SSM = Y'(\frac{1}{n}J_n)Y \).

(a) Show that \( SSR = SSR_M - SSM \).

(b) What are the degrees of freedom associated with \( SSR_M \) and \( SSM \)?

(c) Show that \( SSR_M \) and \( SSM \) are independent of \( SSE \).

(d) Find the distributions of \( SSR_M/\sigma^2 \) and \( SSM/\sigma^2 \).

2. Consider the \( SSR_M \) and \( SSM \) defined in problem #1.

(a) Derive the expected values of the mean squares associated with \( SSR_M \) and \( SSM \).

(b) Show that \( SSR_M \) and \( SSM \) can be used to test the hypotheses \( H_0 : \beta_0 = \beta_1 = \cdots = \beta_{p-1} = 0 \) and \( H_0 : \beta_0 + \beta_1X_1 + \cdots + \beta_{p-1}X_{p-1} = 0 \), respectively. These hypotheses are not particularly interesting. Can you interpret them?

(c) Give test statistics and decision rules for the two hypotheses in part (b).

(d) Give expressions for the power functions.

3. Consider a regression through the origin \( Y_i = \beta_1 X_{i1} + \cdots + \beta_{p-1} X_{i,p-1} + \varepsilon_i \), where \( \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2) \) for \( i = 1, \ldots, n \). (Hint: read section 4.4 in the textbook).

(a) Derive the least square estimator of \( \beta_1 = (\beta_1, \ldots, \beta_{p-1})' \), the fitted values, the hat matrix (denote it by \( H_1 \)) and the residuals in matrix terms.

(b) Show that the sum of the residuals \( \sum_{i=1}^n e_i \) does not necessarily equal to 0 but that \( \sum_{i=1}^n X_{ij}e_i = 0 \) for \( j = 1, \ldots, p - 1 \).

(c) Show that \( \sum Y_i^2 = \sum \hat{Y}_i^2 + \sum e_i^2 \) holds. These sum of squares are called uncorrected sum of squares.

(d) Consider partitioning \( SSTO = SSR_1 + SSE_1 \) where \( SSTO = Y'(I_n - \frac{1}{n}J_n)Y \), \( SSR_1 = Y'(H_1 - \frac{1}{n}J_n)Y \) and \( SSE_1 = Y'(I_n - H_1)Y \). Is \( 0 \leq R^2 \leq 1 \) where \( R^2 = SSR_1/SSTO \)? Justify your answer.

4. 6.6, 6.7, 6.8

5. 6.18(c), 6.19, 6.20, 6.21