

## STANDARD MODEL OF PARTICLE PHYSICS—A HEALTH PHYSICS PERSPECTIVE

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**Abstract**—The Standard Model of Particle Physics is reviewed with an emphasis on its relationship to the physics supporting the health physics profession. Concepts important to health physics are emphasized and specific applications are presented. The capability of the Standard Model to provide health physics relevant information is illustrated with application of conservation laws to neutron and muon decay and in the calculation of the neutron mean lifetime.

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**Key words:** accelerators; beta particles; computer calculations; neutrons

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### INTRODUCTION

THE THEORETICAL formulation describing the properties and interactions of fundamental particles is embodied in the Standard Model of Particle Physics (Bettini 2008; Cottingham and Greenwood 2007; Guidry 1999; Griffiths 2008; Halzen and Martin 1984; Klapdor-Kleingrothaus and Staudt 1995; PDG 2008; Peskin and Schroeder 1995; Quigg 1997). The Standard Model also provides a basis for the unification or consistent description of the strong, electromagnetic, and weak interactions. However, it does not include gravity.

During presentation of the author's health physics certification review courses, a steadily increasing interest in the Standard Model has been noted. Student inquiries usually occur during presentations of pion, muon, and neutron decay characteristics, and discussion of accelerator radiation types. Questions often involve the quark model, contemporary theories such as dark matter or supersymmetry, and the discovery of new particles. These questions arise principally from recent graduates and appear to be influenced by the content of introductory physics and modern physics courses and various

publications describing the operation of the Large Hadron Collider and popular books and articles describing Higgs bosons, magnetic monopoles, supersymmetry particles, dark matter, dark energy, and new particle discoveries (PDG 2008). Many of the students' questions arise from misconceptions regarding the Standard Model and its relationship to the field of health physics (Bevelacqua 2008a). The increasing number and complexity of these questions and misconceptions of the Standard Model motivated the author to write this review article.

The intent of this paper is to present the Standard Model to health physicists in a manner that minimizes the mathematical complexity. This is a challenge because the Standard Model of Particle Physics is a theory of interacting fields. It contains the electroweak interaction (Glashow 1961; Weinberg 1967; Salam 1969) and quantum chromodynamics (QCD) (Gross and Wilczek 1973; Politzer 1973). QCD is a gauge field theory that describes the strong interactions of colored quarks and gluons.

The construction of the Standard Model has been guided by symmetry principles that are supported by group theory. Some of these models and underlying principles may be unfamiliar to health physicists. Accordingly, this paper will address these models and their applications. However, the presentation will omit much of the mathematical rigor required from a theoretical physics perspective and focus on those elements of the Standard Model that are important to health physicists. As part of the discussion, the properties of low-energy particles are introduced and related to the fundamental interactions. These properties are governed by the mediators of the fundamental interactions. The low-energy particles are also characterized in terms of their underlying quark content, and the characteristics of the quarks are reviewed.

An overview of the four fundamental interactions (strong, electromagnetic, weak, and gravitational) and basic conservation laws governing particle interactions is also presented in the context of the Standard Model. This overview provides a foundation for a discussion of the

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characteristics of particle decays and particle interactions, and the resultant radiation types. These characteristics and associated conservation laws are shown to determine which particle decay modes are allowed.

The significance of symmetry is also reviewed. Symmetry principles are illustrated by focusing upon the electromagnetic interaction and the Maxwell equations. Symmetry properties are also reviewed in terms of the field equations and the calculation of particle decay properties. A specific application of the Standard Model is illustrated by considering weak interaction induced neutron decay and by calculating the mean lifetime of the neutron.

### PARTICLE PROPERTIES AND SUPPORTING TERMINOLOGY

In order to ensure the reader is familiar with the background of the Standard Model, basic particle properties are reviewed. Terminology that supports particle characterization and facilitates the discussion is also presented. The properties of particles that are less familiar to some applied health physicists (e.g., muons and kaons) are compared to particles of more familiarity (e.g., neutrons, protons, and electrons).

**Terminology.** Specific terminology is introduced to facilitate presentation of the basic physics associated with the Standard Model. These terms, as utilized in this review, are specific to the Standard Model and include:

- *baryon*—A heavy particle normally composed of three quarks. Protons and neutrons are baryons. Baryons can be electrically charged or uncharged. Standard Model baryons are composed of three quarks;
- *boson*—A particle having integer spin. The mediators or carriers of each of the four fundamental interactions are bosons. The photon,  $W^\pm$  and  $Z^0$ , and gluons are mediators of the electromagnetic, weak, and strong interactions, respectively. Pions are also bosons. Bosons can be electrically charged or uncharged;
- *charge*—A general term used to assign a particular property to a particle or field quanta. Health physicists are most familiar with electric charge that influences processes such as ionization and governs the electromagnetic force. Other types of charge exist including color charge that governs the strong interaction, and weak charge that manifests itself either as a charged or neutral weak current, and these currents govern the weak interaction;
- *fermion*—A particle having half-integer spin. Neutrons, protons, and electrons are examples of fermions. Fermions can be electrically charged or uncharged;
- *flavor*—A designation for the type of quark. The flavors are down (d), up (u), strange (s), charm (c),

bottom (b), and top (t). These designations are further defined in subsequent discussion, sometimes designated by just the first letter of the quark name;

- *generation*—A grouping of quarks and leptons. The Standard Model classifies quarks and leptons into 3 generations. The first generation includes the u and d quarks and the  $e^-$  and  $\nu_e$  and their antiparticles. Second generation particles include the s and c quarks and the  $\mu^-$  and  $\nu_\mu$  and their antiparticles. The third generation includes the b and t quarks and the  $\tau^-$  and  $\nu_\tau$  and their antiparticles;
- *hadron*—A particle that interacts primarily through the strong interaction. Mesons and baryons are hadrons. Hadrons are complex particles having an internal quark structure;
- *lepton*—A fundamental particle that interacts primarily through the weak interaction. The electron and the electron neutrino are examples of leptons. Leptons can be electrically charged or uncharged. Standard model neutrinos are massless. Leptons have no internal structure and include 3 generations;
- *meson*—A middleweight particle normally composed of a quark and an antiquark. The charged and neutral pions are examples of mesons. Standard Model mesons are composed of quark-antiquark pairs; and
- *quark*—A particle having a fractional charge that interacts through the strong, electromagnetic, and weak interactions. Quarks were initially inferred from high-energy electron-proton ( $e-p$ ) scattering. The  $e-p$  scattering cross-section indicates the presence of point-like structures inside the proton that have been interpreted as quarks. The Standard Model incorporates 6 quark flavors and 3 generations.

**Basic particle properties.** Table 1 provides a summary of the properties of selected low-energy particles having a mass below  $2,000 \text{ MeV}\cdot\text{c}^{-2}$  (PDG 2008). These properties include the particle mass, mean lifetime, and dominant decay mode, and are provided for neutrinos (electron, muon, and tau), the electron ( $e^-$ ) and its antiparticle ( $e^+$ ), the muon ( $\mu^-$ ) and its antiparticle ( $\mu^+$ ), the tau ( $\tau^-$ ) and its antiparticle ( $\tau^+$ ), three pions ( $\pi^+$ ,  $\pi^0$ , and  $\pi^-$ ), three kaons ( $K^+$ ,  $K^0$ , and  $K^-$ ), the proton ( $p$ ) and its antiparticle ( $\bar{p}$ ), and the neutron ( $n$ ) and its antiparticle ( $\bar{n}$ ).

Neutrinos are neutral leptons, assumed to be massless in the Standard Model, but experimental data supports a small, but certainly non-zero mass (PDG 2008). There are three known generations of neutrinos ( $\nu$ ) and their corresponding antiparticles (antineutrinos,  $\bar{\nu}$ ). Specifically included are the electron neutrino ( $\nu_e$ ) and its antiparticle ( $\bar{\nu}_e$ ), the muon neutrino ( $\nu_\mu$ ) and its antiparticle ( $\bar{\nu}_\mu$ ), and the tau neutrino ( $\nu_\tau$ ) and its antiparticle ( $\bar{\nu}_\tau$ ). The electron and muon neutrinos are well studied, but

**Table 1.** Properties of selected low energy particles.

Particle	Mass (MeV-c <sup>-2</sup> )	Mean lifetime	Dominant decay mode
$\nu_e$	<0.000002	>300 s/eV <sup>b</sup>	a
$\bar{\nu}_e$	<0.000002	>300 s/eV <sup>b</sup>	a
$\nu_\mu$	<0.19	>15.4 s/eV <sup>b</sup>	a
$\bar{\nu}_\mu$	<0.19	>15.4 s/eV <sup>b</sup>	a
$\nu_\tau$	<18.2	Not yet determined <sup>b</sup>	a,c
$\bar{\nu}_\tau$	<18.2	Not yet determined <sup>b</sup>	a,c
$e^-$	0.511	>4.6 × 10 <sup>26</sup> y	Stable
$e^+$	0.511	>4.6 × 10 <sup>26</sup> y	Stable
$\mu^-$	105.7	2.2 × 10 <sup>-6</sup> s	$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$
$\mu^+$	105.7	2.2 × 10 <sup>-6</sup> s	$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$
$\tau^-$	1777	2.9 × 10 <sup>-13</sup> s	Multiple decay modes
$\tau^+$	1777	2.9 × 10 <sup>-13</sup> s	Multiple decay modes
$\pi^-$	139.6	2.6 × 10 <sup>-8</sup> s	$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
$\pi^0$	135.0	8.4 × 10 <sup>-17</sup> s	$\pi^0 \rightarrow \gamma + \gamma$
$\pi^+$	139.6	2.6 × 10 <sup>-8</sup> s	$\pi^+ \rightarrow \mu^+ + \nu_\mu$
$K^-$	493.7	1.24 × 10 <sup>-8</sup> s	$K^- \rightarrow \mu^- + \bar{\nu}_\mu$
$K^{0d}$	497.6	885.7 s <sup>d</sup>	$K^0 \rightarrow \pi^+ + \pi^-$
$K^+$	493.7	1.24 × 10 <sup>-8</sup> s	$K^+ \rightarrow \mu^+ + \nu_\mu$
$p$	938.3	>2.1 × 10 <sup>29</sup> y	Stable
$\bar{p}$	938.3	>2.1 × 10 <sup>29</sup> y	Stable
$n$	939.6	885.7 s	$n \rightarrow p + e^- + \bar{\nu}_e$
$\bar{n}$	939.6	885.7 s	$\bar{n} \rightarrow \bar{p} + e^+ + \nu_e$

<sup>a</sup> Dependent on the degree of neutrino mixing.

<sup>b</sup> The measured quantities depend upon the Standard Model's mixing parameters and to some extent on the experimental conditions (e.g., energy resolution).

<sup>c</sup> Decay mode not yet determined.

<sup>d</sup> The  $K^0$  particle is a superposition of two states  $K_S^0$  and  $K_L^0$ ;  $K^0 = \frac{1}{\sqrt{2}}(K_S^0 + K_L^0)$  with lifetimes of  $K_S^0 = 8.95 \times 10^{-11}$  s and  $K_L^0 = 5.12 \times 10^{-8}$  s.

much less is known about tau neutrinos. The leptons (electrons, muons, taus, and their associated neutrinos) appearing in Table 1 are fundamental and have no discernible substructure. This is not true of the mesons and baryons that have an underlying quark structure. The properties of these quarks and the composition of selected baryons and mesons are summarized in Tables 2, 3, and 4, respectively.

Table 2 summarizes the properties of quarks within the Standard Model. Both bare and effective masses are provided. Bare quark masses are theoretical values based on an isolated or free quark flavor (PDG 2008). The effective mass or the mass of a quark within a baryon or meson has a different value. These effective mass values are model dependent, include gluon couplings, and may not be experimentally measurable. The results are based on the currently accepted quark interaction spatial dependence (PDG 2008; Griffiths 2008).

The effective masses and particle properties of Table 1 are derived from the interactions defining the Standard Model. It is sufficient to state the electroweak interaction is characterized by SU(2) ⊗ U(1) symmetry, and the strong interaction has SU(3) color symmetry (Bevelacqua 2008a and b; PDG 2008). The

**Table 2.** Properties of quarks within the Standard Model.<sup>a</sup>

Generation	Flavor	Charge (e)	Mass (MeV-c <sup>-2</sup> )	
			Bare <sup>a</sup>	Effective <sup>b</sup>
First	d	$\frac{1}{3}$	3 to 7	340
First	u	$\frac{2}{3}$	1.5–3.0	336
Second	s	$-\frac{1}{3}$	95 ± 25	486
Second	c	$\frac{2}{3}$	1,270	1,550
Third	b	$-\frac{1}{3}$	4,200	4,730
Third	t	$\frac{2}{3}$	171,200	177,000

<sup>a</sup> PDG (2008).

<sup>b</sup> Griffiths (2008).

**Table 3.** Properties of selected baryons within the Standard Model.<sup>a</sup>

Baryon	Quark structure	Charge (e)	Mass (MeV-c <sup>-2</sup> )	Mean lifetime
p	uud	+1	938.3	>2.1 × 10 <sup>29</sup> y
n	udd	0	939.6	885.7 s
$\Lambda$	uds	0	1,115.6	2.63 × 10 <sup>-10</sup> s
$\Sigma^+$	uus	+1	1,189.4	8.02 × 10 <sup>-11</sup> s
$\Sigma^0$	uds	0	1,192.5	7.40 × 10 <sup>-20</sup> s
$\Sigma^-$	dds	-1	1,197.4	1.48 × 10 <sup>-10</sup> s
$\Xi^0$	uss	0	1,314.9	2.90 × 10 <sup>-10</sup> s
$\Xi^-$	dss	-1	1,321.7	1.64 × 10 <sup>-10</sup> s

<sup>a</sup> Derived from PDG (2008).

**Table 4.** Properties of selected mesons within the Standard Model.<sup>a</sup>

Meson	Quark structure	Charge (e)	Mass (MeV-c <sup>-2</sup> )	Mean lifetime
$\pi^+$	$u\bar{d}$	+1	139.6	2.6 × 10 <sup>-8</sup> s
$\pi^-$	$d\bar{u}$	-1	139.6	2.6 × 10 <sup>-8</sup> s
$\pi^0$	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	135.0	8.4 × 10 <sup>-17</sup> s
$K^+$	$u\bar{s}$	+1	493.7	1.24 × 10 <sup>-8</sup> s
$K^0$	$d\bar{s}$	0	497.6	<sup>b</sup>
$K^-$	$s\bar{u}$	-1	493.7	1.24 × 10 <sup>-8</sup> s

<sup>a</sup> Derived from PDG (2008).

<sup>b</sup> See Table 1.

electroweak interaction includes the weak interaction and the electromagnetic interaction includes the Maxwell field equations that have U(1) symmetry. The U and SU designations refer to unitary and special unitary groups, respectively. The numbers in parenthesis are the dimensionality of the groups.

Tables 3 and 4 provide the properties of selected Standard Model SU(3) baryons and mesons, respectively. The quark structure, electric charge, mass, and lifetime are provided. It should be noted that other

symmetry group structures are possible for models that include physics beyond the Standard Model (PDG 2008). These alternative models are not a significant consideration within the context of this review.

The particle's charge can be derived from the internal quark structures of Tables 3 and 4. In addition, the lifetime values are readily derived from the Standard Model. Subsequent discussion will illustrate the neutron mean lifetime calculation.

As an illustration of the success of the Standard Model, the quark charge results of Table 2 and specific quark structures of Tables 3 and 4 can be used to predict the charge of a particle. For example, the proton has a uud structure (Table 3) with a resulting charge given by the algebraic sum of the individual quark charges (i.e.,  $2/3 e + 2/3 e - 1/3 e = e$ ) and the  $K^+$  structure is  $\bar{u}s$  (Table 4), which provides the expected charge (i.e.,  $2/3 e + 1/3 e = e$ ).

## FUNDAMENTAL INTERACTIONS

Four fundamental interactions or forces describe the phenomena observed in the universe. These are the strong, electromagnetic, weak, and gravitational interactions, and their properties are summarized in Table 5. The unique aspects of the strong, weak, and electromagnetic interactions govern particle decays and interactions, which influence the health physics consequences of the resulting radiation types.

The field boson is the mediator or the carrier of the force. For example, the electromagnetic interaction is mediated by photons. A photon is exchanged between the two particles involved in an electromagnetic interaction. The field mediators have been directly observed or inferred from observed phenomena. All mediators are based on significant experimental evidence with the exception of the graviton that is inferred from gravitational field theory.

These field bosons give the various fundamental interactions unique properties. Although the photon is a

well-known radiation type, it has a much deeper physical significance because its exchange defines the electromagnetic interaction. In a similar fashion, the exchange of gluons (of which there are 8) defines the strong interaction. The weak interaction is also complex because there are three particles (i.e.,  $W^+$ ,  $W^-$ , and  $Z^0$ ) that are exchanged. Properties of the field bosons give each fundamental interaction a distinctive character. For example, the weak interaction governs beta decay, and manifests itself in the magnitude and dose profile of the neutrino effective dose (Bevelacqua 2004, 2008a). The field boson mass also exhibits a distinctive nature.

The photon, gluons, and graviton are all massless. In contrast, the weak interaction field bosons have masses in the 80–92 GeV-c<sup>-2</sup> range. The field boson mass does not uniquely determine the nature of the fundamental interaction. It is the collective nature of the field boson's mass, charge, number of allowed states, lifetime, and coupling constant that determines the unique characteristics of an interaction.

In Table 5, the source of the interaction refers to the basic physical quantity that gives rise to the force. The four fundamental interactions arise from very different physical constructs. For example, the gravitational and electromagnetic interactions are derived from mass and electric charge, respectively. The concepts of mass and electric charge are well known to health physicists. However, weak charge and color charge are not.

It is well known from classical physics that a moving charge produces a current (Jackson 1999). Therefore, weak charges in motion generate a weak current. Weak currents produce weak forces that govern lepton interactions. Leptons have no color charge, and consequently do not participate in the strong interaction. Neutrinos have no electric charge so they experience no electromagnetic force, but they do participate in the weak interaction.

Color charge produces the strong interaction. However, color charge is considerably more complex than

**Table 5.** Fundamental interactions and their properties.

Property	Fundamental interaction			
	Gravitational	Electromagnetic	Weak	Strong
Field bosons	Graviton	Photon	$W^+$ , $W^-$ , and $Z^0$	8 gluons
Mass of field boson (GeV-c <sup>-2</sup> )	0	0	$M_W = 80.398$ $M_Z = 91.1876$	0
Range of the interaction (m)	$\infty$	$\infty$	$10^{-18}$	$\leq 10^{-15}$
Source of the interaction	Mass	Electric charge	Weak charge	Color charge
Strength (relative to the strong interaction)	$10^{-39}$	$10^{-2}$	$10^{-5}$	1
Typical cross-section (m <sup>2</sup> )	<sup>a</sup>	$10^{-33}$	$10^{-39b}$	$10^{-30}$
Typical lifetime (s)	<sup>a</sup>	$10^{-20}$	$10^{-10}$	$10^{-23}$

<sup>a</sup> In view of the range and source of the gravitational interaction, the cross-section and lifetime are not well-defined quantities.

<sup>b</sup> This cross-section is applicable to contemporary accelerator energies (Bevelacqua 2008a).

electric charge. Color charge is a property assigned to a quark or gluon, and it has three states (i.e., red, blue, and green). There are eight gluons governing the strong interaction instead of one photon for the electromagnetic interaction. Since the gluons themselves carry a color charge, they can directly interact with other gluons. This possibility is not available with the electromagnetic force since photons do not have electric charge. Therefore, it is not surprising that the strong and electromagnetic forces have different characteristics.

In Table 5, the interaction strength is a measure of the magnitude of the force as measured over its effective range. The term interaction strength is intrinsically ambiguous because it depends on the measurement distance from the source. Accordingly, the strength values listed in Table 5 may be quoted with different values by other authors (Bettini 2008; Cottingham and Greenwood 2007; Griffiths 2008; Halzen and Martin 1984). Table 5 provides the strength relative to the strong interaction. In terms of decreasing strength, nature orders these interactions as follows: strong, electromagnetic, weak, and gravitational. As noted previously, the Standard Model does not include the gravitational interaction.

The cross-section describes the probability of a typical interaction that is solely governed by one of the fundamental interactions. The lifetime represents the time over which an interaction occurs assuming the interaction is governed solely by that fundamental force. For example, strong interactions typically create particles with cross-sections in the mb ( $10^{-27}$  cm<sup>2</sup>) range that have lifetimes on the order of  $10^{-23}$  s. The cross-section and lifetime of a created particle are often clear indications of the type of force involved in an interaction.

Neutrino interaction cross-sections, governed by the weak interaction, are orders of magnitude smaller than the typical strong or electromagnetic interaction cross-sections (PDG 2008). The weak interaction cross-section magnitude makes neutrino detection difficult for energies encountered at contemporary facilities (Bevelacqua 2004, 2008a).

The gravitational interaction is an interaction affecting massive objects such as planets, solar systems, and galaxies. The terms cross-section and lifetime are not clearly defined within the context of the gravitational interaction. Since the gravitational interaction is not included in the Standard Model, no further commentary is provided.

### Fundamental interactions and their health physics impacts

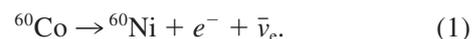
The strong interaction binds quarks into mesons and baryons, and is responsible for binding nucleons within

the nucleus. It arises from the exchange of gluons between quarks and governs a number of commonly observed processes including fission, fusion, and activation. The radiation hazards from these processes are well known to health physicists.

The electromagnetic force results from the exchange of photons. It governs much of the physics encountered in our daily lives. For example, atomic physics and molecular chemistry are governed by the electromagnetic interaction. This interaction also influences nuclear reactions and competes with the strong force in nuclear processes. The electromagnetic interaction depends on the electric charge of the interacting particles. As a practical example, ions can be accelerated because they have an electric charge and the electromagnetic force governs their final energy.

The weak force governs processes such as beta decay and positron decay. Weak interactions also dictate the behavior of leptons.

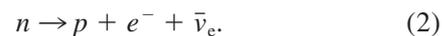
Although the fundamental interactions are distinct phenomena, they often appear collectively in nature. As an example, consider the beta decay of <sup>60</sup>Co:



The nuclear energy levels in the <sup>60</sup>Co and <sup>60</sup>Ni nuclei are determined by the strong and electromagnetic interactions. The relative position of the energy levels in the <sup>60</sup>Co and <sup>60</sup>Ni nuclei, their specific properties (e.g., spin and parity), and conservation laws determine if the transition between a specific set of energy levels produces a beta particle.

During beta decay, a neutron single particle level in <sup>60</sup>Co (<sup>59</sup>Co + n) transitions to a proton single particle level in <sup>60</sup>Ni (<sup>59</sup>Co + p) with the emission of an electron (beta particle) and antielectron neutrino.

From a nuclear transformation perspective, beta decay is described by:



Within the Standard Model, beta decay is described as a sequential process incorporating the W<sup>-</sup> boson:



Although eqns (1) through (4) represent the same physical process, they differ in terms of the type of model utilized in the description of the neutron decay process.

Eqns (1) through (4) may be accepted on face value, but the reader should question why these are the physical beta decay modes. The Standard Model provides insight into this question.

## CONSERVATION LAWS

Fundamental physics is governed by basic symmetries that are expressed in terms of a set of conservation laws that permit certain reactions and forbid others. In this section, the specific conservation laws that facilitate an understanding of the processes that lead to radiation types of concern in health physics are examined. Four conservation laws are useful in understanding the underlying physics of the Standard Model that is relevant to health physics applications. Other conservation laws (e.g., energy, linear momentum, and angular momentum) are also applicable, but this review focuses on the laws specifically related to the Standard Model. These conservation laws include:

1. *Conservation of Electric Charge*: All three of the fundamental interactions governing health physics applications (strong, electromagnetic, and weak) conserve electric charge. Many particles participating in the various fundamental interactions contain electric charge (e.g., protons, pions, muons, and electrons);
2. *Conservation of Color Charge*. The electromagnetic and weak interactions do not affect color charge. Color charge is conserved in strong interactions. Physical particles (e.g., baryons and mesons) are colorless. This means mesons contain a quark of one color (red, blue, or green) and an antiquark of the same anticolor (antired, antiblue, or antigreen). Baryons consist of three quarks each of a different color;
3. *Conservation of Baryon Number*. The total number of quarks is a constant. Since baryons are composed of three quarks, the baryon number is just the quark number divided by 3. There is no corresponding conservation of meson number since the mesons, composed of quark-antiquark pairs, carry zero baryon number; and
4. *Conservation of Electron Number, Muon Number, and Tau Number*. The strong interaction does not affect leptons. In a pure electromagnetic interaction, the same particle comes out (accompanied by a photon) as went in. The weak interaction only mixes together leptons from the same generation. Therefore, the lepton number, muon number, and tau number are all conserved.

These conservation laws provide a key input to understanding the decay schemes summarized in Table 1. An examination of the health physics consequences of particle decays and their associated radiation types is possible when these conservation laws are combined with an understanding of the Standard Model of Particle Physics.

For example, leptons interact primarily through the weak interaction and electrically charged leptons experience the effects of the electromagnetic force. They are

not affected by the strong interaction. There are six leptons, classified according their electric charge ( $Q$ ), electron number ( $L_e$ ), muon number ( $L_\mu$ ), and tau number ( $L_\tau$ ). The leptons are naturally grouped into three families or generations as summarized in Table 6.

There are also six antileptons, with all the signs in Table 6 reversed (i.e., + to - and - to +). The positron for example has an electric charge of +1 and an electron number of -1. Considering both particles and antiparticles, there are 12 leptons in the Standard Model.

In a similar manner, there are six flavors of quarks (u, d, s, c, b, and t) (PDG 2008), with their quantum numbers classified according to their electric charge, upness (U), downness (D), strangeness (S), charm (C), bottomness (B), and topness (T). These labels are historical and have no underlying physical meaning. The quarks also fall into three generations as summarized in Table 7. Again, all signs are reversed on a table of antiquarks. Since each quark and antiquark comes in three colors, there are 36 distinct quarks in the Standard Model.

Table 5 and the subsequent discussion noted 8 mediators for the strong interaction (gluons), the photon for the electromagnetic interaction, and 3 mediators for the weak interaction ( $W^+$ ,  $W^-$ , and  $Z^0$ ). This yields 12 mediators for the Standard Model.

A careful reader notes that one of the shortcomings of the Standard Model is the number of free parameters or elementary particles that it requires: 12 leptons, 36 quarks, and 12 mediators. There is also at least one other particle (the Higgs boson). Therefore, there is a minimum of 61 parameters to address. The Standard Model has been remarkably successful, but mounting evidence [e.g., indication that neutrinos have mass and recent publications regarding four quark mesons and five quark baryons (PDG 2008)] suggests that physics beyond the Standard Model is required to explain these and other results. However, the Standard Model can provide results of relevance to health physics applications.

**Consequences of the conservation laws and the Standard Model.** With knowledge of conservation laws and the Standard Model, we will illustrate how these

**Table 6.** Lepton classification.<sup>a</sup>

Generation	Lepton	Charge (e)	$L_e$	$L_\mu$	$L_\tau$
First	$e^-$	-1	1	0	0
First	$\nu_e$	0	1	0	0
Second	$\mu^-$	-1	0	1	0
Second	$\nu_\mu$	0	0	1	0
Third	$\tau^-$	-1	0	0	1
Third	$\nu_\tau$	0	0	0	1

<sup>a</sup> Derived from PDG (2008).

**Table 7.** Quark classification.<sup>a</sup>

Generation	Quark	Charge (e)	D	U	S	C	B	T
First	d	$-\frac{1}{3}$	-1	0	0	0	0	0
First	u	$\frac{2}{3}$	0	1	0	0	0	0
Second	s	$-\frac{1}{3}$	0	0	-1	0	0	0
Second	c	$\frac{2}{3}$	0	0	0	1	0	0
Third	b	$-\frac{1}{3}$	0	0	0	0	-1	0
Third	t	$\frac{2}{3}$	0	0	0	0	0	1

<sup>a</sup> Derived from PDG (2008).

laws are satisfied for neutron decay and muon decay. Tables 8 and 9 summarize beta decay and muon decay, respectively. The decays of various particles summarized in Table 1 are not arbitrary, and are governed by the conservation laws that follow from the symmetries underlying the Standard Model. In particular, decay modes of the neutron and muon are governed by the conservation of baryon number, lepton number (for each generation), electric charge, and color charge. Other decay modes (e.g.,  $\mu^- \rightarrow n + e^-$ ,  $\tau^- \rightarrow n + \mu^-$ , and  $n \rightarrow p + e^-$ ) are excluded by these conservation laws.

Tables 8 and 9 illustrate the application of conservation laws to predict a particle’s decay and its associated radiation types. These laws and the Standard Model are sufficient to predict the radiation types that occur in particle decay and interaction processes of interest in health physics applications. An examination of the initial and final states of Tables 8 and 9 illustrate the impact of conservation laws on allowed processes.

Conservation laws are also implied by the fundamental interactions and their underlying symmetry properties. Noether’s Theorem (Bevelacqua 2008a; Cottingham and Greenwood 2007; Griffiths 2008) provides a mathematical proof of the relationship between a given symmetry and its conservation law. Within the Standard Model, symmetries are expressed

**Table 8.** Beta decay ( $n \rightarrow p + e^- + \bar{\nu}_e$ ).

Conservation law	Initial state	Final state		
	$n$	$p$	$e^-$	$\bar{\nu}_e$
Baryon number	1	1	0	0
Lepton number ( $L_e$ ) <sup>a</sup>	0	0	1	-1
Lepton number ( $L_\mu$ ) <sup>a</sup>	0	0	0	0
Electric charge	0	e	-e	0
Color charge	0	0	0	0

<sup>a</sup> See Table 6.

**Table 9.** Muon decay ( $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$ ).

Conservation law	Initial state	Final state		
	$\mu^-$	$e^-$	$\nu_\mu$	$\bar{\nu}_e$
Baryon number	0	0	0	0
Lepton number ( $L_e$ ) <sup>a</sup>	0	1	0	-1
Lepton number ( $L_\mu$ ) <sup>a</sup>	1	0	1	0
Electric charge	-e	-e	0	0
Color charge	0	0	0	0

<sup>a</sup> See Table 6.

in terms of group properties. As an example, the electromagnetic, weak, and strong interaction field quanta are represented by the generators of the unitary group of dimension 1 [U(1)], the special unitary group of dimension 2 [SU(2)], and the special unitary group of dimension 3 [SU(3)], respectively.

Within the Standard Model, the number of generators ( $N$ ) of a group of dimension  $n$  ( $n > 1$ ) is given by:

$$N = n^2 - 1. \tag{5}$$

For  $n = 1$ , there is a single generator. These generators are equivalent to the field bosons summarized in Table 5. Therefore, it is expected that the electromagnetic ( $n = 1$ ), weak ( $n = 2$ ), and strong ( $n = 3$ ) interactions have 1, 3, and 8 field bosons, respectively. This prediction is observed experimentally with one field boson (photon) for the electromagnetic interaction, 3 field bosons ( $W^+$ ,  $W^-$ , and  $Z^0$ ) for the weak interaction, and 8 field bosons (8 gluons) for the strong interaction. The prediction of the number and characteristics of the field bosons for the electromagnetic, weak, and strong interactions is an impressive success of the Standard Model of Particle Physics, and provides additional confidence in its ability to predict the radiation types and their intensity resulting from the decay and interaction of fundamental particles. An additional illustration of the relationship between symmetry groups and underlying physics is provided by considering the U(1) group.

**Electrodynamics and U(1) symmetry**

The symmetry groups become more complex as their dimension increases. The simplest group involved in the Standard Model is the U(1) group describing electrodynamics. U(1) is defined in terms of a single, real, and continuous parameter that is usually selected to be a rotation angle. The U(1) group can be applied to the electromagnetic interaction embodied in the Maxwell equations.

The symmetrized form of the Maxwell equations is invariant under a duality transformation (Jackson 1999).

For example, the electric ( $\vec{E}$ ) and magnetic ( $\vec{H}$ ) fields are related by (Bevelacqua 2008b; Jackson 1999):

$$\begin{aligned}\vec{E} &= \vec{E}' \cos \xi + \vec{H}' \sin \xi \\ \vec{H} &= -\vec{E}' \sin \xi + \vec{H}' \cos \xi,\end{aligned}\quad (6)$$

where  $\xi$  is a rotation angle and the primed variables are defined with respect to the coordinate system generated by this rotation. The form of eqn (6) is often an indication of the underlying U(1) symmetry group.

In general, symmetries are related to specific physical properties when they follow from conservation laws. The conservation of energy, linear and angular momentum, baryon number, and lepton number are examples. Similar relationships are derived by considering the higher order symmetries involved in the Standard Model. For example, baryons and mesons can be classified in terms of SU(3) group representations including the singlet, octet, and decuplet structures (Griffiths 2008; PDG 2008).

The next section of this paper provides a subset of the defining equations of the Standard Model. These equations further illustrate the relationship between symmetry and predicted physics.

### STANDARD MODEL FORMALISM

As with many field theories, the Standard Model can be formulated in terms of a Lagrangian density (Bettini 2008; Cottingham and Greenwood 2007; Griffiths 2008; Halzen and Martin 1984). The Lagrangian density ( $L$ ) is written in terms of the fields  $\varphi_i$  and their derivatives  $\partial_\mu \varphi_i$ , where

$$\partial_\mu \varphi_i \equiv \frac{\partial \varphi_i}{\partial x^\mu}, \quad (7)$$

where  $i$  labels the field and  $\mu = 0, 1, 2,$  and  $3$  labels the coordinates (i.e.,  $\mu = 0$  is the time coordinate and  $\mu = 1, 2,$  and  $3$  labels the spatial coordinates). The Lagrangian density is important because the specific field equation (e.g., Klein-Gordon for spin 0 particles, Dirac for spin 1/2, or Proca for spin 1) is obtained from the Euler-Lagrange equations (Bettini 2008; Cottingham and Greenwood 2007; Griffiths 2008; Halzen and Martin 1984):

$$\frac{\partial L}{\partial \varphi_i} - \partial_\mu \left[ \frac{\partial L}{\partial (\partial_\mu \varphi_i)} \right] = 0. \quad (8)$$

The complete Standard Model Lagrangian density (Cottingham and Greenwood 2007; PDG 2008) is quite detailed and includes electroweak and QCD components. For the purpose of this paper, only a portion of the Lagrangian density relevant to a subsequent neutron decay example is provided without a detailed description of the mathematical relationships comprising each term. The general characteristics of

the various terms are noted to illustrate the complexity of the Standard Model machinery. This approach is consistent with the desire to minimize the mathematical complexity while maximizing health physics relevance. The notation and definition of terms of Cottingham and Greenwood (2007) are used in representing the Lagrangian density.

Eqn (9) illustrates how the components of the Lagrangian density relate to the neutron decay calculation:

$$\begin{aligned}L &= \frac{1}{2} \partial_\mu h \partial^\mu h - m^2 h^2 - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{4} \varphi_0^2 (g_1^2 + g_2^2) Z_\mu Z^\mu \\ &\quad - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{2} [(D_\mu W_\nu^+)^* - (D_\nu W_\mu^+)^*] [D^\mu W^{+\nu} \\ &\quad - D^\nu W^{+\mu}] + \frac{1}{2} g_2^2 \varphi_0^2 W_\mu^- W^{+\mu}, \quad (9)\end{aligned}$$

where  $h$  is the Higgs field,  $m$  is the mass of the considered particle,  $Z_\mu$  is the vector field of the  $Z^0$  boson,  $Z_{\mu\nu}$  is a field strength tensor associated with the  $Z^0$  boson,  $W_\mu^-$  is the vector field of the  $W^-$  boson,  $W_\mu^+$  is the vector field of the  $W^+$  boson,  $g_1$  and  $g_2$  are coupling constants, and  $\varphi_0$  is the vacuum expectation value of the Higgs field. The  $D_\mu$  operator and  $A_{\mu\nu}$  tensor terms are defined in terms of vector fields and their derivatives, and an angle defined by the masses of the weak interaction field bosons.

As an illustration of symmetries inherent in eqn (9), consider three specific parameters. These are a rotation angle determined by the ratio of weak mediator masses, and two specific vector quantities ( $A_\mu$  and  $Z_\mu$ ). Specifically, let  $\theta_w$  be a rotation angle (Weinberg angle) defined by:

$$\cos \theta_w = \frac{M_W}{M_Z}. \quad (10)$$

An inherent symmetry of the Standard Model is illustrated by introducing two vector fields ( $B_\mu$  and  $W_\mu^3$ ) (Cottingham and Greenwood 2007). The  $B_\mu$  and  $W_\mu^3$  fields can be rewritten in terms of  $\theta_w$  in a manner that is analogous to eqn (6) based on U(1) symmetry:

$$B_\mu = A_\mu \cos \theta_w - Z_\mu \sin \theta_w \quad (11)$$

$$W_\mu^3 = A_\mu \sin \theta_w + Z_\mu \cos \theta_w. \quad (12)$$

In eqns (11) and (12),  $\theta_w$  is defined by the  $Z$  and  $W$  boson masses following eqn (10). This specific relationship can be contrasted with the U(1) electromagnetic example that noted the rotation angle  $\xi$  was dependent on the specific formulation of the Maxwell equations (Bevelacqua 2008a). However, the similarity of eqns 6, 11, and 12 is illustrative of the types of symmetries encountered in the Standard Model.

Symmetry also influenced the predictions of the Standard Model. For example, the discovery of the  $W^+$ ,  $W^-$ , and  $Z^0$  bosons and an accurate determination of their

masses verified the Standard Model's prediction of the  $\theta_w$  value. The verification of this key prediction further established the validity of the Standard Model (PDG 2008) and its assumed symmetry characteristics.

### NEUTRON DECAY AND MEAN LIFETIME CALCULATION

Neutron decay is a fundamental problem in both theoretical physics and health physics. A recent theoretical effort (Faber et al. 2009) provided a rigorous Standard Model calculation of the neutron mean lifetime. Details of this calculation are well beyond the scope of this review. However, the basic elements of a Standard Model calculation of the neutron mean lifetime are outlined in subsequent discussion.

The previous sections of this review provided the basis for a calculation of the neutron lifetime based on the Standard Model and the characteristics of the neutron and associated decay particles. To illustrate the essential elements of the underlying physics, four neutron decay lifetime cases (I, II, III, and IV) are presented.

The Case I calculation uses eqns (3) and (4) to include the  $W^-$  boson and its subsequent decay into an electron ( $e^-$ ) and antielectron neutrino ( $\bar{\nu}_e$ ). In Case I, the neutron ( $n$ ) and proton ( $p$ ) are treated as point particles. With a point particle assumption, a longer neutron mean lifetime is expected because no internal correlations are included in the description of the neutron and proton. These correlations would enhance the neutron decay rate.

A more physical calculation (Case II) is achieved by including weak charge coupling as part of the  $W^-$  interactions of eqns (3) and (4). Case III improves the Case II calculation with the inclusion of specific quark structures into eqn (3). Following Table 3, the neutron has a ddu quark structure and the proton has a uud structure. In eqn (3), the ud quark pair acts as spectators to the quark transition:

$$d \rightarrow u + W^- \quad (13)$$

Charge conservation is met because the d quark has a charge of  $-1/3 e$  and u has a charge of  $+2/3 e$  (see Tables 2 and 3).

Case IV includes a radiative correction based on a detailed, theoretical analysis (Faber et al. 2009).

**Calculational approach.** The neutron mean lifetime is obtained using Feynman calculus (Halzen and Martin 1984; Peskin and Schroeder 1995; Bettini 2008; Griffiths 2008) to calculate the amplitude ( $M$ ). The amplitude is an important quantity from a health physics perspective. Once it is determined, relevant health physics quantities (e.g., mean lifetime, branching ratios, and cross-sections) are readily determined.

For a given particle (1) that decays into several other particles 2, 3, 4, ... k, the decay rate ( $\Gamma$ ) is determined from the amplitude and kinematic factors according to the relationship:

$$\Gamma = \frac{S}{2\hbar m_1} \int |M|^2 (2\pi)^4 \times \delta^4(p_1 - p_2 - p_3 - \dots - p_k) \prod_{j=2}^k 2\pi \times \delta(p_j^2 - m_j^2 c^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}, \quad (14)$$

where  $m_i$  is the mass of the  $i^{\text{th}}$  particle,  $p_i$  is its four-momentum,  $S$  is a statistical factor, and  $\theta(p_j^0)$  is the Heaviside step function. This step function [ $\theta(z)$ ] is 0 if  $z < 0$  and is 1 if  $z > 0$ . The Dirac delta function ( $\delta$ ) is the derivative of the Heaviside step function.

For neutron decay, the particles (label number, associated four-momentum) are: n (1,  $p_1$ ),  $\bar{\nu}_e$  (2,  $p_2$ ), p (3,  $p_3$ ),  $e^-$  (4,  $p_4$ ), and  $W^-$  (5,  $q$ ). The mean lifetime ( $\tau$ ) is the reciprocal of the decay rate. In subsequent mean lifetime calculations, detailed derivations are not presented, but salient results of the Feynman calculus are quoted.

**Neutron mean lifetime—Case I.** Using Feynman calculus, the Case I total decay rate expression is:

$$\Gamma_1 = \frac{1}{4\pi^3 \hbar} \left( \frac{g_w}{2M_W c^2} \right)^4 (m_e c^2)^5 \times \left[ \frac{1}{15} (2x^4 - 9x^2 - 8) \sqrt{x^2 - 1} + x \ln(x + \sqrt{x^2 - 1}) \right], \quad (15)$$

where

$$x = \frac{m_n - m_p}{m_e} \quad (16)$$

In eqn (16),  $m_n$ ,  $m_p$ , and  $m_e$  are the neutron, proton, and electron masses, respectively. This result is stated without providing the intermediate calculation steps that do not affect the health physics purpose of this paper. Using the values in the PDG (2008) leads to the Case I neutron mean lifetime ( $\tau$ ):

$$\tau_1 = \frac{1}{\Gamma_1} = 1315.7 \text{ s}. \quad (17)$$

As expected from previous discussion, the calculated mean lifetime exceeds the experimental value of 885.7 s

(PDG 2008). However, improved results are expected for subsequent cases that refine the simple point particle model of Case I.

**Neutron mean lifetime—Case II.** Case I assumes that the proton and neutron are point particles that interact with the  $W^-$  in the same way that leptons interact. However, this is a poor assumption because the proton and neutron are composite quark structures (see Table 3).

To account for the net coupling strength of the composite nucleon structure and the associated gluon interactions, weak charge is introduced into eqn (3) through coupling coefficients (PDG 2008). The inclusion of the weak coupling coefficients enhances the decay by a factor of 1.4587 that leads to an improved mean lifetime of:

$$\tau_{II} = \frac{1}{\Gamma_{II}} = \frac{1315.7 \text{ s}}{1.4587} = 902.0 \text{ s.} \quad (18)$$

**Neutron mean lifetime—Case III.** Case III includes the enhancements of Case II, and accounts for the  $d$  to  $u$  quark transition through the  $W^-$  boson noted in eqns (3), (4), and (13). The enhancement of Case III leads to an amplitude that is similar to Case II with the addition of a factor dependent on the Cabibbo angle ( $\theta_C$ ) (Cabibbo 1963). Electroweak interactions account for the decay properties of quarks by including functions of the Cabibbo angle that has a value of  $13.04^\circ$  (PDG 2008).

The net effect of including the  $d \rightarrow u$  quark transition is to retard the neutron decay by a factor of  $\cos^{-2}(\theta_C)$  and leads to a Case III mean lifetime of:

$$\tau_{III} = \frac{1}{\Gamma_{III}} = \frac{902.0 \text{ s}}{\cos^2(\theta_C)} = 950.4 \text{ s.} \quad (19)$$

The careful reader will note that the more physically correct result of Case III yields a poorer result than Case II. When such occurrences are encountered, they are an indication that the calculation is not yet complete and is omitting some essential physics.

**Neutron mean lifetime—Case IV.** A missing element in the Case I, II, and III calculations is the radiative corrections (e.g., higher order Feynman loop diagrams) (Gudkov et al. 2005) that specify details of the quark-gluon interaction. These interactions alter the spatial distribution of quarks within a nucleon and enhance the decay rate but require a mathematical treatment well beyond the scope of this review.

A rigorous theoretical physics calculation would include radiative correlations and the details of the electron spectrum that could be parameterized in terms of

scalar, vector, and tensor couplings. These factors have recently been included in a neutron decay calculation by Faber et al. (2009). This calculation includes the aforementioned effects, including a 3.9% correction for radiative effects, and calculates a neutron mean lifetime value essentially in agreement with data.

Case IV is defined by modifying the Case III results with radiative corrections. Including the aforementioned 3.9% radiative correction, the Case IV mean neutron lifetime is obtained from the Case III lifetime:

$$\tau_{IV} = \frac{\tau_{III}}{1.039} = \frac{950.4 \text{ s}}{1.039} = 914.7 \text{ s,} \quad (20)$$

which is about 3% larger than the measured value of 885.7 s. This result is sufficient to illustrate the capability of the Standard Model to solve a problem relevant to health physics.

The interested reader is referred to Faber et al. (2009) for additional details of their Standard Model calculation. Details of the types of calculations involved in Cases I, II, and III are found in textbooks describing Feynman calculus (Halzen and Martin 1984; Peskin and Schroeder 1995; Bettini 2008; Griffiths 2008).

## CONCLUSION

The Standard Model of Particle Physics provides a theoretical methodology for describing processes relevant to health physics applications including the interaction characteristics of fundamental particles. This review provided specific applications explaining the decay modes of the neutron and muon in terms of conservation laws and illustrated the relationship of symmetry to the formulation of the Standard Model.

A simplified calculation using the Standard Model predicted a neutron mean lifetime about 3% larger than the measured value. Detailed Standard Model calculations yield results in agreement with experiments. These results suggest that the Standard Model yields credible predictions relevant to health physics applications.

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